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EXAMPLES FOR PRACTICE IN THE USE OF
SEVEN-FIGURE LOGARITHMS.



EXAMPLES FOR PRACTICE

IN THE USE OF

SEVEN-FIGURE

LOGARITHMS

For the Use of Schools and Colleges

BY

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1888

PREFATORY NOTE.

CONSIDERABLE practice is required by most students before they acquire the accuracy which is absolutely necessary in the use of logarithms. According to my experience, mistakes are more frequently made by taking out a wrong logarithm than by blundering in the arithmetic. It would seem therefore that a student should always be expected to take out himself all the logarithms, whether of numbers or of Trigonometrical Functions, which are needed in the solution of any question; and that the practice, which has been so common, of supplying him with all these in addition to the data should be abandoned. It is mainly with the wish of bringing about this result that I have put together the accompanying collection of examples. As the answers are given in each case, this little book is the result of a very considerable amount of work. I have taken all pains to secure accuracy, but cannot hope to have completely succeeded, and shall be grateful to anyone who will point out to me any mistakes I may have made.

I have given what may be thought a disproportionate number of examples of the calculation of the parts of a tetrahedron when

the lengths of its edges are given. This was done, chiefly because I am convinced that no other kind of examples so soon teaches the habitual accuracy which ought to be attained, but also to some extent to give examples for the use of those who have occasion for practice in the solution of Spherical Triangles. In the calculation of each tetrahedron is involved the obtaining all the parts of four Spherical Triangles, and this has induced me, after considerable hesitation, to abstain from giving explicit examples for such solutions.

I have principally, but not exclusively, used Chambers' Mathematical Tables in my working out of these examples. Of course the seventh significant place in numbers, and the second decimal place in seconds of angles, are not absolutely to be relied upon. If this book should be used with tables of six or five-figure logarithms, the corresponding number of places should be struck off my results, the usual correction being made when necessary in the remaining final figure.

COOPER'S HILL COLLEGE, *Sept. 4th*, 1888.

EXAMPLES FOR PRACTICE IN THE USE OF SEVEN-FIGURE LOGARITHMS

I.—Calculation of $L \sin$, $L \tan$, etc., when the angle is given, and conversely.

If it is required to find the $L \sin$, $L \cos$, $L \tan$, ..., of an angle not exactly given in the Tables, it is best to take out from the Tables the $L \sin$, $L \cos$, or $L \tan$ of that angle in the Tables which is next below the proposed angle; and then to calculate the "proportional part" for the seconds by a small sum in Practice worked at the side: this proportional part being then added (with the proper sign) to the logarithm taken out. Thus to calculate $L \tan 28^\circ 40' 49.2''$, $L \sin 27^\circ 45' 43.54''$, $L \cos 54^\circ 57' 58.2''$.

$\begin{array}{r} L \tan 28^\circ 40' = 9.7377714 \\ \text{diff. for } 49.2'' = \quad 2461 \\ \hline L \tan 28^\circ 40' 49.2'' = 9.7380175 \end{array}$	$\begin{array}{r} 60'' = 3001 \\ 48'' - 1'' = 2400 \\ 1.2'' = 1\frac{1}{5} \times 60 = 60 \\ \hline 2461 \end{array}$
$\begin{array}{r} L \sin 27^\circ 45' = 9.6680265 \\ \text{diff. for } 43\ 54'' = \quad 1742 \\ \hline L \sin 27^\circ 45' 23\ 54'' = 9.6682007 \end{array}$	$\begin{array}{r} 60'' = 2400 \\ 42'' = 1'' \times 1680 \\ 1.5'' = \frac{3}{4} \times 60 = 60 \\ .04'' = 1\frac{1}{2} \times 60 = 1 \\ \hline 1742 \end{array}$
	$\begin{array}{r} L \cos 54^\circ 57' = 9.7591321 \\ \text{diff. for } 58.2'' = \quad 1746 \\ \hline 9.7589575 \end{array}$

Each of these examples might also be done by multiplying in each case the number of seconds by the quotient obtained by dividing by 60 the difference for 60". Thus, in the last example, this quotient is 30 and $58.2 \times 30 = 1746$. When the difference for 60" contains only three figures, this method is generally the shorter one. When the $L \sin$, $L \cos$, $L \tan$, ..., of an angle is given, and the angle is required, look out in the Tables the $L \sin$ (or $L \cos$ or $L \tan$, ...) *nearest* to the given one, whether greater or less, and write this under the given one, *omitting* those figures which are the same in both. Then take out the corresponding angle (putting + or - after it according as the number of seconds due to the difference is to be added or subtracted), and

the difference for 60"; and calculate the number of seconds by the usual proportion. Thus having given—

$$(1) \quad \begin{array}{r|l} L \sin x = 9.8775429 & 1100 \\ 48^\circ 58' - \} & \frac{601}{172} \end{array} \quad \begin{array}{l} 10320(9.38", \text{ or } x=48^\circ 57' 50.62"; \\ 42 \\ 90 \end{array}$$

$$(2) \quad \begin{array}{r|l} L \cos x = 9.7076837 & 2130 \\ 59^\circ 20' - \} & \frac{064}{773} \end{array} \quad \begin{array}{l} 46380(21.78, \text{ or } x=59^\circ 19' 38.22"; \\ 378 \\ 165 \\ 159 \end{array}$$

$$(3) \quad \begin{array}{r|l} L \sec x = 10.1324081 & 1158 \\ 42^\circ 30' + \} & \frac{3691}{390} \end{array} \quad \begin{array}{l} 25400(18.48, \text{ or } x=42^\circ 30' 18.48"; \\ 9820 \\ 556 \\ 928 \end{array}$$

The method of "proportional parts" must not be employed for the $L \sin$ or $L \tan$ of an angle less than $3^\circ 30'$, the methods for "small" angles being then employed; as also for the $L \cos$ or $L \sec$ of an angle greater than $86^\circ 30'$.

[Not many examples for working are given under this heading, as every example in all the subsequent sections will furnish one or more.]

EXAMPLES. I.

1. Find the $L \sin$ of the following angles :—

$$68^\circ 47' 41.5", \quad 73^\circ 21' 23.91", \quad 78^\circ 29' 43.85", \quad 139^\circ 21' 10.74".$$

$$[\text{Ans. } 9.9695517, \quad 9.9814136, \quad 9.9911858, \quad 9.8138458.]$$

2. Find the $L \cos$ of the following angles :—

$$25^\circ 45' 33.61", \quad 22^\circ 24' 19.04", \quad 18^\circ 49' 46.32", \quad 66^\circ 59' 38.97.$$

$$[\text{Ans. } 9.9545452, \quad 9.965912, \quad 9.9761128, \quad 9.9919823.]$$

3. Find the $L \tan$ of the following angles :—

$$23^\circ 48' 41.12", \quad 46^\circ 0' 43.79", \quad 66^\circ 11' 18.88", \quad 43^\circ 59' 16.21".$$

$$[\text{Ans. } 9.6447248, \quad 10.0153473, \quad 10.3552752, \quad 9.9846527.]$$

4. Find the value of x from each of the following equations :—

$$(a) \quad L \sin x = 9.5740674, \quad L \sin x = 9.8091259, \quad L \sin x = 9.8281920, \quad L \sin x = 9.9860122;$$

$$(\beta) \quad L \tan x = 9.3600730, \quad L \tan x = 9.8353762, \quad L \tan x = 10.0562416, \quad L \tan x = 10.1321843;$$

$$(\gamma) \quad L \cos x = 9.9670225, \quad L \cos x = 9.8610172, \quad L \cos x = 9.7544211, \quad L \cos x = 9.3701332.$$

$$[\text{Ans. } \left\{ \begin{array}{l} (a) \quad 22^\circ 1' 34.47", \quad 40^\circ 7' 2.6", \quad 42^\circ 19' 13.01", \quad 75^\circ 32' 9.83"; \\ (\beta) \quad 12^\circ 54' 18.65", \quad 34^\circ 23' 30.51", \quad 48^\circ 41' 58.59", \quad 53^\circ 35' 16.23"; \\ (\gamma) \quad 21^\circ 51' 1.23", \quad 43^\circ 26' 12.02", \quad 59^\circ 22' 56.98", \quad 76^\circ 26' 17.35". \end{array} \right.]$$

5. Find the values of the following fractions :—

$$(1) \quad \frac{\sin 58^\circ 9' 20.9'' \sin 49^\circ 26' 47.92'' \sin 65^\circ 18' 24.84''}{\sin 57^\circ 11' 25.7'' \sin 53^\circ 17' 18.06'' \sin 60^\circ 29' 59.54''}$$

$$(2) \quad \frac{\sin 57^\circ 11' 25.7'' \sin 61^\circ 20' 39.56'' \sin 68^\circ 33' 21.04''}{\sin 58^\circ 9' 20.9'' \sin 57^\circ 30' 9.46'' \sin 73^\circ 21' 46.38''}$$

$$(3) \quad \frac{\sin 61^\circ 24' 17.1'' \sin 57^\circ 30' 9.46'' \sin 60^\circ 29' 59.54''}{\sin 53^\circ 56' 29.5'' \sin 61^\circ 20' 39.56'' \sin 65^\circ 18' 24.84''}$$

$$(4) \quad \frac{\sin 53^\circ 56' 29.5'' \sin 53^\circ 17' 18.06'' \sin 73^\circ 21' 46.38''}{\sin 61^\circ 24' 17.1'' \sin 49^\circ 26' 47.92'' \sin 68^\circ 33' 21.04''}$$

[Each fraction=1.]

6. Prove the truth of the equations:—

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} = 2 \sqrt{\frac{\sin s \sin(s-a) \sin(s-b) \sin(s-c)}{\sin a \sin b \sin c}}, \quad (2s = a + b + c),$$

for the following systems:—

$$\begin{aligned} (1) & \left\{ \begin{array}{l} a = 27^{\circ} 28' 41.02'', \quad A = 49^{\circ} 24' 1.48'' \\ b = 33^{\circ} 18' 34.24'', \quad B = 64^{\circ} 38' 45.10'' \\ c = 35^{\circ} 42' 29.32'', \quad C = 73^{\circ} 49' 49.80'' \end{array} \right\}; \quad (2) \left\{ \begin{array}{l} a = 50^{\circ} 0' 30.76'', \quad A = 49^{\circ} 24' 1.48'' \\ b = 62^{\circ} 59' 51.52'', \quad B = 62^{\circ} 0' 25.14'' \\ c = 81^{\circ} 9' 58.50'', \quad C = 101^{\circ} 40' 57.08'' \end{array} \right\}; \\ (3) & \left\{ \begin{array}{l} a = 81^{\circ} 17' 39.16'', \quad A = 77^{\circ} 15' 8.76'' \\ b = 66^{\circ} 19' 31.38'', \quad B = 64^{\circ} 38' 45.10'' \\ c = 97^{\circ} 2' 17.90'', \quad C = 101^{\circ} 40' 57.08'' \end{array} \right\}; \quad (4) \left\{ \begin{array}{l} a = 65^{\circ} 31' 27.26'', \quad A = 77^{\circ} 15' 8.76'' \\ b = 55^{\circ} 29' 1.08'', \quad B = 62^{\circ} 0' 25.14'' \\ c = 63^{\circ} 39' 57.86'', \quad C = 73^{\circ} 49' 49.80'' \end{array} \right\}; \end{aligned}$$

also prove that the value of each member of the equations is (1) 1.64556, (2) .991042, (3) .9867222, (4) 1.071654.

II.—Calculation of $\sqrt{a^2 + b^2}$, when a, b are given numbers.

Taking u to denote $\sqrt{a^2 + b^2}$, then if θ be an angle determined by the equation $\tan \theta = \frac{b}{a}$, $u = a \sec \theta = b \operatorname{cosec} \theta$. The value of u may be determined from either of these equations; if both be used, the calculation is thereby checked. The work may be conveniently arranged as in the following example.

$a = 47993.27$ $b = 38932.89$	$\log b = 4.5903167$ $\log a = 4.6811804$ <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $L \tan \theta = 9.9091363$ $39^{\circ} 3' - \}$ $2582 \}$ <div style="text-align: right; padding-right: 10px;">440 77</div>	$4620(1.79$ 2038 <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> 2306 $\theta = 39^{\circ} 2' 58.21''$	$\log \sec \theta = .1098046$ <div style="text-align: right; padding-right: 10px;">.31</div> <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $\log a = 4.6811804$ <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $\log u = 4.7909819$	$1025 = 60''$ <div style="text-align: right; padding-right: 10px;">1.8 = $\frac{1}{10} = 30$</div> <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> <div style="text-align: right; padding-right: 10px;">.01 = —</div> <div style="text-align: right; padding-right: 10px;">31</div>	$\left. \begin{array}{l} 8 \\ 2 \end{array} \right\}$			
<table style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding-right: 10px;"> $\log \operatorname{cosec} \theta = .2006606$ <div style="text-align: right; padding-right: 10px;">+46</div> </td> <td style="border-left: 1px solid black; padding-left: 10px;"> $1558 = 60''$ <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $1.8 = 46$ <div style="text-align: right; padding-right: 10px;">7</div> </td> </tr> <tr> <td style="padding-right: 10px;"> <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $\log b = 4.5903167$ <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $\log u = 4.7909819$ </td> <td style="border-left: 1px solid black; padding-left: 10px;"> <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> <div style="text-align: right; padding-right: 10px;">.01 = —</div> <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> <div style="text-align: right; padding-right: 10px;">46</div> </td> </tr> </table>					$\log \operatorname{cosec} \theta = .2006606$ <div style="text-align: right; padding-right: 10px;">+46</div>	$1558 = 60''$ <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $1.8 = 46$ <div style="text-align: right; padding-right: 10px;">7</div>	<hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $\log b = 4.5903167$ <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> $\log u = 4.7909819$	<hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> <div style="text-align: right; padding-right: 10px;">.01 = —</div> <hr style="border: 0; border-top: 1px solid black; margin: 2px 0;"/> <div style="text-align: right; padding-right: 10px;">46</div>
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The two calculations agree in giving $\log u = 4.7909819$, which is therefore almost certainly correct. This gives $u = 61799.07$.

EXAMPLES. II.

DATA.	ANSWERS.
1. $a = 571.4284, \quad b = 428.5713.$	714.2854 ($\theta = 36^{\circ} 52' 11.65''$).
2. $a = 93921.58, \quad b = 37917.43.$	101286.7 ($\theta = 21^{\circ} 59' 4.87''$).
3. $a = 4285.714, \quad b = 5714.285.$	7142.855 ($\theta = 36^{\circ} 52' 11.65''$).
4. $a = 23727.92, \quad b = 7873.11.$	25000.01 ($\theta = 18^{\circ} 21' 22.5''$).
5. $a = 7139.247, \quad b = 3143.892.$	7800.827 ($\theta = 23^{\circ} 46' 1.68''$).
6. $a = 52791.39, \quad b = 37387.41.$	64689.64 ($\theta = 35^{\circ} 18' 23.46''$).
7. $a = 171428.4, \quad b = 169999.8.$	241428.3 ($\theta = 44^{\circ} 45' 36.92''$).
8. $a = 3081.708, \quad b = 212.7818.$	3089.046 ($\theta = 3^{\circ} 56' 59.34''$).
9. $a = 9853.08, \quad b = 2127.818.$	10080.22 ($\theta = 12^{\circ} 11' 10.14''$).
10. $a = 13790.4, \quad b = 13790.3.$	19502.5 ($\theta = 44^{\circ} 59' 59.26''$).
11. $a = 803.761, \quad b = 803.76.$	1136.689 ($\theta = 44^{\circ} 59' 59.88''$).

EXAMPLES. II.—*Continued.*

DATA.		ANSWERS.	
12. $a = 7118.12$,	$b = 606.13$.	7143.878	$(\theta = 4^{\circ} 52' 1.82")$.
13. $a = 34285.68$,	$b = 9999.99$.	35714.25	$(\theta = 16^{\circ} 15' 36.75")$.
14. $a = 7372.46$,	$b = 6989.37$.	10158.95	$(\theta = 43^{\circ} 28' 19.35")$.
15. $a = 1185.184$,	$b = 345.6796$.	1234.567	$(\theta = 16^{\circ} 15' 36.88")$.
16. $a = 273.0004$,	$b = 135.9992$.	305.	$(\theta = 26^{\circ} 28' 51.17")$.
17. $a = 272.9$,	$b = 130.2022$.	305.0008	$(\theta = 26^{\circ} 31' 24.3")$.
18. $a = 5488.42$,	$b = 2273.378$.	5940.623	$(\theta = 22^{\circ} 30')$.
19. $a = 378.4125$,	$b = 487.4775$.	615.37	$(\theta = 52^{\circ} 5' 4.83")$.
20. $a = 27891.83$,	$b = 36789.17$.	46167.06	$(\theta = 52^{\circ} 49' 56.27")$.
21. $a = 262.087$,	$b = 151.316$.	302.632	$(\theta = 29^{\circ} 59' 59.98")$.
22. $a = 56471.9$,	$b = 97812.2$.	112943.8	$(\theta = 60^{\circ})$.
23. $a = 8055.837$,	$b = 5924.819$.	10000	$(\theta = 36^{\circ} 20')$.
24. $a = 7601.54$,	$b = 6497.43$.	10000	$(\theta = 40^{\circ} 31' 20")$.
25. $a = 7968.81$,	$b = 23695.95$.	25000	$(\theta = 71^{\circ} 24' 45.02")$.
26. $a = 1236.113$,	$b = 716.1294$.	1428.572	$(\theta = 30^{\circ} 57' 5")$.
27. $a = 74511.63$,	$b = 19109.62$.	76923.095	$(\theta = 14^{\circ} 23' 3.6")$.
28. $a = 4313.546$,	$b = 3999.524$.	5882.351	$(\theta = 42^{\circ} 50' 9.6")$.
29. $a = 4127.817$,	$b = 1365.548$.	4347.825	$(\theta = 18^{\circ} 18' 18.17")$.
30. $a = 437.7493$,	$b = 796.7571$.	909.0909	$(\theta = 61^{\circ} 12' 54")$.
31. $a = 6151.67$,	$b = 1104.292$.	6250.001	$(\theta = 10^{\circ} 10' 36.6")$.
32. $a = 4761.903$,	$b = 13468.55$.	14285.67	$(\theta = 70^{\circ} 31' 42.88")$.

III.—Solution of Equations of the form $a \cos x + b \sin x = c$.

The two values of x which satisfy the equation $a \cos x + b \sin x = c$ being denoted by α, β , $\frac{1}{a} \cos \frac{1}{2}(\alpha + \beta) = \frac{1}{b} \sin \frac{1}{2}(\alpha + \beta) = \frac{1}{c} \cos \frac{1}{2}(\alpha - \beta)$. First find $\frac{1}{2}(\alpha + \beta)$ from the equation $\tan \frac{1}{2}(\alpha + \beta) = \frac{b}{a}$, and then $\frac{1}{2}(\alpha - \beta)$ from either or both of the equations $\cos \frac{1}{2}(\alpha - \beta) = \frac{c}{a} \cos \frac{1}{2}(\alpha + \beta) = \frac{c}{b} \sin \frac{1}{2}(\alpha + \beta)$. Writing the equation so that c is positive, then if a be negative and b positive, put $x = \pi - y$; if a be positive and b negative, put $x = -y$; and if both a and b be negative, put $x = \pi + y$. We shall thus always have an equation in which the coefficients are all positive. Arrange the work according to the following pattern.

$$\begin{array}{r|l}
 \begin{array}{l}
 a=27839 \\
 b=29273 \\
 c=28565
 \end{array} & \begin{array}{l}
 14.4664672\dots (10 + \log b) \\
 - 4.4445536 \dots (\log a) \\
 \hline
 L \tan \alpha + \beta = 10.0218136 \\
 \begin{array}{l}
 46^\circ 26' + \\
 253\phi
 \end{array} \left\{ \begin{array}{l}
 7380 \\
 756 \times 6\phi \\
 4536 (17.93) \\
 2006 \\
 235 \\
 73
 \end{array} \right.
 \end{array}
 \end{array}
 \quad
 \begin{array}{r|l}
 \begin{array}{l}
 9.8383441 \\
 - 397 \\
 4.4558342 \\
 - 4.4445536
 \end{array} & \begin{array}{l}
 9.8494850 \\
 = L \cos \frac{1}{2}(\alpha - \beta) \\
 = L \cos 45^\circ
 \end{array}
 \end{array}
 \quad
 \begin{array}{r|l}
 (1) & \begin{array}{l}
 1329-60'' \\
 18-398 \\
 - .06-1 \\
 - .01=-
 \end{array}
 \end{array}
 \quad
 \begin{array}{r|l}
 (2) & \begin{array}{l}
 1201=60''=20\frac{1}{8} \times 60 \\
 358.6 \\
 3
 \end{array}
 \end{array}$$

Thus $\frac{a+\beta}{2} = 46^{\circ}26'17.93''$, $\frac{a-\beta}{2} = 45^{\circ}$, } whence $a = 91^{\circ}26'17.93''$, $\beta = 1^{\circ}26'17.93''$.

If the values of a, b, c be such that $c^2 > a^2 + b^2$, the values of x will be impossible. Should such data be given, the value found for $I. \cos \frac{1}{2}(a - \beta)$ would be greater than 10, and the corresponding angle impossible.

EXAMPLES. III.

DATA.	ANSWERS.
1. $a = 11175.3$, $b = 5600.24$, $c = 10548.25$.	$a = 59^{\circ}4'$, $\beta = 5^{\circ}50'$.
2. $a = 58533.32$, $b = 49909.95$, $c = 62137.14$.	$a = 76^{\circ}34'24.7''$, $\beta = 4^{\circ}20'$.
3. $a = 67313.97$, $b = 58129.13$, $c = 62721.45$.	$a = 85^{\circ}57'38.3''$, $\beta = -4^{\circ}20'9.6''$.
4. $a = 39713.76$, $b = 29254.24$, $c = 18397.98$.	$a = 104^{\circ}28'34.01''$, $\beta = -31^{\circ}43'24.07''$.
5. $a = 6342.667$, $b = 3914.217$, $c = 6549.328$.	$a = 66^{\circ}11'27.25''$, $\beta = 3^{\circ}10'7.07''$.
6. $a = 549.1238$, $b = 508.1741$, $c = 618.3272$.	$a = 77^{\circ}2'50.77''$, $\beta = 8^{\circ}30'59.71''$.
7. $a = 35.67812$, $b = 28.73407$, $c = 40.13598$.	$a = 67^{\circ}40'1.8''$, $\beta = 10^{\circ}1'36''$.
8. $a = 7891.293$, $b = 889.312$, $c = 7926.027$.	$a = 9^{\circ}58'39.75''$, $\beta = 2^{\circ}52'55.13''$.
9. $a = 3712.484$, $b = 1139.115$, $c = 3795.726$.	$a = 29^{\circ}14'59.21''$, $\beta = 4^{\circ}51'56.79''$.
10. $a = 801.3982$, $b = 116.4593$, $c = 869.154$.	$a = 8^{\circ}28'58.52''$, $\beta = 6^{\circ}54'58.52''$.
11. $a = 25130.85$, $b = 23846.16$, $c = 27389.27$.	$a = 81^{\circ}15'31.62''$, $\beta = 6^{\circ}23'20.84''$.
12. $a = 1790.009$, $b = 2811.935$, $c = 2956.161$.	$a = 85^{\circ}2'25.2''$, $\beta = 30''$.
13. $a = 78913.47$, $b = 59134.72$, $c = 67314.95$.	$a = 83^{\circ}47'49.92''$, $\beta = -10^{\circ}6'15.32''$.
14. $a = 3581.72$, $b = 3312.93$, $c = 4645.07$.	$a = 60^{\circ}34'48.14''$, $\beta = 24^{\circ}57'17.40''$.
15. $a = 428.5714$, $b = 571.4286$, $c = 714.2857$.	$a = 53^{\circ}48.35''$, $\beta = 53^{\circ}7'48.35''$.
16. $a = 20785.2$, $b = 20611.99$, $c = 20058.41$.	$a = 89^{\circ}2'13.18''$, $\beta = 0^{\circ}29'0.70''$.
17. $a = 9999.79$, $b = 8654.33$, $c = 11235.84$.	$a = 72^{\circ}42'19.56''$, $\beta = 9^{\circ}2'37.46''$.
18. $a = 37912.37$, $b = 28776.52$, $c = 39148.21$.	$a = 71^{\circ}51'49.02''$, $\beta = 2^{\circ}32'7.22''$.
19. $a = 84513.27$, $b = 9837.12$, $c = 79125.64$.	$a = 28^{\circ}12'10.98''$, $\beta = -14^{\circ}55'28.70''$.
20. $a = 831.2798$, $b = 415.6399$, $c = 888.8931$.	$a = 44^{\circ}25'31.66''$, $\beta = 8^{\circ}22'32.54''$.
21. $a = 813908.5$, $b = 600305.1$, $c = 1000000$.	$a = 45^{\circ}0'0.04''$, $\beta = 27^{\circ}49'18.4''$.
22. $a = 7933.533$, $b = 6087.614$, $c = 9914.449$.	$a = 45''$, $\beta = 30''$.
23. $a = 7261.248$, $b = 6875.629$, $c = 9915.867$.	$a = 50^{\circ}52'30''$, $\beta = 36^{\circ}0'0''$.
24. $a = 35170.77$, $b = 35538.95$, $c = 48363.02$.	$a = 60''$, $\beta = 30^{\circ}35'48''$.
25. $a = 786.1347$, $b = 563.9187$, $c = 803.4125$.	$a = 69^{\circ}31'40.06''$, $\beta = 1^{\circ}46'41.84''$.
26. $a = 43759.16$, $b = 24189.58$, $c = 44576.71$.	$a = 92^{\circ}0'0''$, $\beta = -34^{\circ}8'0.00''$.
27. $a = 137904$, $b = 137903$, $c = 195025$.	$a = 44^{\circ}59'59.24''$, $\beta = 44^{\circ}59'59.24''$.
28. $a = 803.76$, $b = 803.761$, $c = 1136.689$.	$a = 45^{\circ}0'0.12''$, $\beta = 45^{\circ}0'0.12''$.
29. $a = 70$, $b = 24$, $c = 37$.	$a = 78^{\circ}55'28.72''$, $\beta = -41^{\circ}4'31.28''$.
30. $a = 70$, $b = 24$, $c = 64.08587$.	$a = 48^{\circ}55'28.72''$, $\beta = -11^{\circ}4'31.28''$.
31. $a = 39405.98$, $b = 17622.89$, $c = 21583.55$.	$a = 21^{\circ}54'18.56''$, $\beta = 95^{\circ}54'18.6''$.
32. $a = 97.8122$, $b = 56.4719$, $c = 112.9438$.	$a = 30''$, $\beta = 30''$.
33. $a = 27442$, $b = 11366.89$, $c = 29703.12$.	$a = 22^{\circ}30'$, $\beta = 22^{\circ}30'$.
34. $a = 5488.43$, $b = 273.382$, $c = 5940.635$.	$a = 22^{\circ}30'$, $\beta = 22^{\circ}30'$.
35. $a = 43155.16$, $b = 68971.5$, $c = 71859.22$.	$a = 85^{\circ}55'55''$, $\beta = 30''$.
36. $a = 395.76$, $b = 228.48$, $c = 323.12$.	$a = 75^{\circ}0'3.18''$, $\beta = -15^{\circ}0'12.68''$.
37. $a = 193.19$, $b = 198.79$, $c = 196.01$.	$a = 90^{\circ}49'6.6''$, $\beta = 0^{\circ}49'6.6''$.
38. $a = 27.839$, $b = 28.273$, $c = 28.565$.	$a = 89^{\circ}23'44.83''$, $\beta = 1^{\circ}29'25.83''$.
39. $a = 239.9$, $b = 3992.1$, $c = 3568.9$.	$a = 113^{\circ}23'13.59''$, $\beta = 59^{\circ}44'5.71''$.
40. $a = 11.599$, $b = 49.121$, $c = 35.689$.	$a = 121^{\circ}42'50.56''$, $\beta = 31^{\circ}42'50.56''$.
41. $a = 3496.7$, $b = 63497$, $c = 5125.7$.	$a = 106^{\circ}41'30.68''$, $\beta = -16^{\circ}0'18.28''$.
42. $a = 34967$, $b = 63497$, $c = 51257$.	$a = 106^{\circ}9'32.31''$, $\beta = 16^{\circ}9'32.31''$.

IV.—Calculation of $\sqrt{b^2+c^2-2bc \cos A}$.

Denoting the expression by u , its value may be calculated by any one of the three following methods :—

$$(1) u_1 = (b+c) \cos \theta, \text{ where } \sin \theta = \frac{2\sqrt{bc} \cos \frac{1}{2}A}{b+c};$$

$$(2) u_2 = (b-c) \sec \theta, \text{ where } \tan \theta = \frac{2\sqrt{bc}}{b-c} \sin \frac{1}{2}A; \quad (b > c)$$

$$(3) u_3 = (b+c) \sin \frac{1}{2}A \sec \theta, \text{ where } \tan \theta = \frac{b-c}{b+c} \cot \frac{1}{2}A;$$

The calculation of one example by each of these three methods is subjoined.

$b=12369.12$
 $c=4123.04$
 $A=40^{\circ}55'55''$
 $b+c=16492.16$
 $b-c=8246.08$
 $A=20^{\circ}27'57.5''$
 2
 $=20^{\circ}28' 2.5''$

DATA.

$\log b=4.0923388$
 $\log c=3.6152175$
 $2)7.7075563$
 $\log \sqrt{bc}=3.8537782$
 $\log (b+c)=4.2172775$
 $\log (b-c)=3.9162475$

(1)

$.30103$ 3.8537782 9.9716820 $19)$ $4 \ 2172775$	471 $2.4-18 \ 8$ $.1- \ 7$ $2.5-19 \ 5$	$\log \cos \theta=1.7667739$ 432 $\log (b+c)=4.2172775$ $\log u_1=3.9840946$ 05 9640.3 $9- \ 41$ $u_1=9640.390$	1753 $10-292 \ 2$ $4.8-140 \ 2$ $.03-1 \ 2$ $14.84=4.32$
--	--	--	--

$L \sin \theta=9.9092146$
 $54^{\circ}14'- \ 371$
 $910 \ 225$

$)1350(14.84''$
 440
 76
 32

(2)

$.3010300$ 3.8537782 9.5436489 $-141)$ 3.9162475	3386 $2 \ 2113$ $.5- 28$ $2.5-141$	$\log \sec \theta=.0678489$ -16 $\log (b-c)=3.9162475$ $\log u_2=3.9840948$ $u_2=9640.394$	765 $1.2 =15 \ 3$ $.02= \ 2$ $15 \ 5$
--	---	--	--

$L \tan \theta=9.7821955$
 $31^{\circ}12'- \ 2013$
 $2851 \ 58$

$)3480(1.22$
 629
 588

(3)

3.9162475 10.4280331 $161)$ 4.2172775	3858 $2 =128 \ 6$ $.5= 32 \ 2$ $2.5=161$	$\log \sec \theta=.2232324$ $-498)$ $\log \sin \frac{1}{2}A=1.5436348$ $\log (b+c)=4.2172775$ $\log u_3=3.9840949$ $u_3=9640.396$	1693 $15 =423 \ 3$ $2 = 56 \ 4$ $.66= 18 \ 6$ $17.66=498$
--	---	--	---

$L \tan \theta=10.1270192$
 $53^{\circ}16'- \ 968$
 $2636 \ 776$

$)46560(17.66$
 2020
 1748
 1664

The three values of $\log u$, found according to the three methods, will generally have slight differences in the last place. In the examples given, the answers will exhibit the three values of u , denoted by u_1, u_2, u_3 ; and the mean of these is taken for the value of u . Thus, in the one calculated, $u_1 = 9640.390, u_2 = 9640.394, u_3 = 9640.396, u = 9640.393$.

EXAMPLES. IV.

DATA.	ANSWERS.
1. $b=45123.76, c=34179.24, A=24^\circ 57' 18.6''$.	$u_1=20193.17, u_2=u_3=20193.20, u=20193.18$.
2. $b=2357.126, c=2075.352, A=67^\circ 31' 55''$.	$u_1=2474.689, u_2=u_3=2474.691, u=2474.69$.
3. $b=2357.126, c=2075.382, A=67^\circ 31' 55''$.	$u_1-u_2=u_3-u=2474.705$.
4. $b=47392.16, c=25497.83, A=56^\circ 37' 55.2''$.	$u_1=39583.89, u_2=u_3=39853.92, u=39853.9$.
5. $b=38914.86, c=34819.73, A=120^\circ$.	$u_1=63888.85, u_2=63888.91, u_3=63888.89,$ $u=63888.88$.
6. $b=73869.13, c=9347.25, A=60^\circ$.	$u_1=69667.42, u_2=u_3=69667.40, u=69667.41$.
7. $b=543.2971, c=276.8139, A=36^\circ 55' 57.6''$.	$u_1=362.4475, u_2=362.4477, u_3=362.447,$ $u=362.4474$.
8. $b=47394.91, c=21382.75, A=26^\circ 39' 45''$.	$u_1=29869.01, u_2=u_3=29869.03, u=29869.02$.
9. $b=7984.637, c=4138.363, A=31^\circ 37' 52.8''$.	$u_1=4961.009, u_2=u_3=4961.012, u=4961.011$.
10. $b=73182.2, c=47297.4, A=39^\circ 19' 5.62''$.	$u_1=47297.42, u_2=u_3=47297.39, u=47297.4$.
11. $b=670.344, c=580.551, A=120^\circ$.	$u_1-u_2=u_3-u=1080.309$.
12. $b=3791.298, c=413.115, A=4^\circ 30'$.	$u_1=3379.611, u_2=3379.69, u_3=3379.612,$ $u=3379.611$.
13. $b=3789.371, c=1134.199, A=23^\circ 6' 16.8''$.	$u_1=2781.982, u_2=u_3=2781.98, u=2781.981$.
14. $b=890.8505, c=534.1539, A=53^\circ 7' 48.38''$.	$u_1=718.6850, u_2=718.6852(=u), u_3=718.6855$.

Other examples for practice may be taken from VII.

V.—Triangles in which are given two angles and one side.

Let a be the given length of one side; then whichever two of the three angles are given, the third is at once found from the equation $A+B+C=180^\circ$; then find R the radius of the circumscribed circle from the equation $2R = a/\sin A$, and the other sides b, c from the equations $b = 2R \sin B, c = 2R \sin C$. The radii of the inscribed and escribed circles r, r_1, r_2, r_3 , if required, may best be found from the equations $r = (s-a) \tan \frac{A}{2}, r_1 = s \tan \frac{A}{2}, r_2 = (s-c) \cot \frac{A}{2}, r_3 = (s-b) \cot \frac{A}{2}$, or the corresponding system involving $\tan \frac{B}{2}$ or $\tan \frac{C}{2}$, taking that one of the three half angles which is nearest to 45° . The equation $r_1 + r_2 + r_3 - r = 4R$ should then be employed to check the calculations. An example is worked out at full length below.

DATA.		
$a=27837.93$	$\log a = 4.4446369$	$\frac{277}{-}$
$A=77^\circ 36' 57.6''$	$-\log \sin A = -1.9897766$	$2.4 = 100 = 11 \quad 1$
$B=54^\circ 22' 12.48''$		
	$\log 2R = 4.4548614$	
hence $C=48^\circ 0' 49.92''$	28501.09	01
		13

$\log 2R = 4.4548614$ $\log \sin B = 1.9099634$ <hr/> $\log b = 4.3648436$ <hr/> $23165.60.$ <hr/> 113	905 $12'' = 181$ $.48 = 7 \quad 2$ <hr/> $\log \tan \frac{1}{2}A = 9.9052672$ <hr/> $\log \tan \frac{1}{2}A = 1.9053914$ <hr/> $\log \cot \frac{1}{2}A = 0.0946086$	$\log 2R = 4.4548614$ $\log \sin C = 1.8710735$ <hr/> $\log c = 4.3260295$ <hr/> $21185.05.$ <hr/> 10	1137 $48 = 909 \quad 6$ $1.8 = 34 \quad 1$ $.12 = 2 \quad 3$ <hr/> $\log s - a = 3.9167886$ $\log s - b = 4.1115545$ $\log s - c = 4.1734555$ $\log s = 4.5574385$
$a = 27837.93$ $b = 23165.60$ $c = 21185.05$ <hr/> $2s = 72188.58$ <hr/> $s = 36094.29$	$s - a = 8256.36$ $s - b = 12928.69$ $s - c = 14909.24$ <hr/> $s = 36094.29$	$\frac{A}{2} = 38^\circ 48' 28.8''$ <hr/> $L \tan \frac{1}{2}A = 9.9052672$ <hr/> 1242 <hr/> $30' = 1293 \quad 5$ $-1.2'' = -51 \quad 7$	<hr/> $\log s - a = 3.9167886$ $\log s - b = 4.1115545$ $\log s - c = 4.1734555$ $\log s = 4.5574385$
$\log s - a = 3.9167886$ $\log \tan \frac{1}{2}A = 1.9053914$ <hr/> $\log r = 3.8221800$ <hr/> $6640.182.$ <hr/> -12	$\log s = 4.5574385$ $\log \tan \frac{1}{2}A = 1.9053914$ <hr/> $\log r_1 = 4.4628299$ <hr/> $29028.85.$ <hr/> 128	$\log s - c = 4.1734555$ $\log \cot \frac{1}{2}A = 0.0946086$ <hr/> $\log r_2 = 4.2686061$ <hr/> $18538.05.$ <hr/> 12	$\log s - b = 4.1115545$ $\log \cot \frac{1}{2}A = 0.0946086$ <hr/> $\log r_3 = 4.2061631$ <hr/> $16075.45.$ <hr/> 121

ANSWERS.

$C = 48^\circ 0' 49.92''$	$r = 6640.18.$
$b = 23165.66.$	$r_1 = 29028.85.$
$c = 21185.05.$	$r_2 = 18538.05.$
$R = 14250.54.$	$r_3 = 16075.45.$

CHECK.

$-r = -6640.18$
$r_1 = 29028.85$
$r_2 = 18538.05$
$r_3 = 16075.45$
$4R = 57002.17$
$R = 14250.54$

The distance between the circumcentre, orthocentre, and incentre are to be found, when required, from the equations $\Delta_1^2 = R^2 - 2Rr$, $\Delta_2^2 = R^2 + 2\rho^2$, $\Delta_3^2 = \rho^2 + 2r^2$, where $\rho^2 = -4R^2 \cos A \cos B \cos C$.

EXAMPLES. V.

DATA.	ANSWERS.
1. $a=7$ $B=38^\circ 36'$ $C=129^\circ 23'$	$A=12^\circ 1'$ $b=20.97616$ $c=25.98715$ $R=16.81107$ $r=2.103095$ $r_1=2.839854$ $r_2=9.448844$ $r_3=57.05867$
2. $a=86602.54$ $B=80^\circ 42' 17.3''$ $C=39^\circ 17' 42.7''$	$A=60^\circ$ $b=98686.93$ $c=63331.6$ $R=50000$ $r=21770.725$ $r_1=71771.02$ $r_2=105618.62$ $r_3=44381.38$
3. $a=91035.91$ $B=37^\circ 43'$ $C=66^\circ 43' 41.9''$	$A=65^\circ 33' 18.1''$ $b=61175.72$ $c=91864.15$ $R=50000$ $r=20788.34$ $r_1=88339.88$ $r_2=41683.78$ $r_3=90764.7$

EXAMPLES. V.—Continued.

DATA.

ANSWERS.

4. $a=96840.39$ $B=37^{\circ}43'$ $C=66^{\circ}43'28.3''$	$A=75^{\circ}33'31.7''$ $b=61175.71$ $c=91861.56$ $A'=50000$	$r=21779.2$ $r_1=96840.57$ $r_2=42674.64$ $r_3=82264$
5. $a=11935.26$ $B=41^{\circ}24'34.3''$ $C=24^{\circ}17'42.3''$	$A=114^{\circ}17'43.4''$ $b=8661.392$ $c=5387.97$ $A'=6547.5$	$r=1636.927$ $r_1=20119.62$ $r_2=4910.528$ $r_3=2796.707$
6. $a=22733.78$ $B=90^{\circ}$ $C=67^{\circ}30'$	$A=22^{\circ}30'$ $b=59406.23$ $c=54884.19$ $A'=29703.115$	$r=9105.87$ $r_1=13627.91$ $r_2=68512.1$ $r_3=45778.32$
7. $a=6347.287$ $A=78^{\circ}33'14.7''$ $B=39^{\circ}59'43.5''$	$C=61^{\circ}27'1.8''$ $b=4162.347$ $c=6404.116$ $A'=3238.04$	$r=1725.267$ $r_1=6916.216$ $r_2=2510.033$ $r_3=5251.178$
8. $a=12791.38$ $A=62^{\circ}37'15.5''$ $B=68^{\circ}25'37.24''$	$C=48^{\circ}57'7.26''$ $b=13395.9$ $c=10863.65$ $A'=7202.49$	$r=3487.813$ $r_1=11208.31$ $r_2=12596.28$ $r_3=8433.18$
9. $a=7674.893$ $A=97^{\circ}18'47.1''$ $C=40^{\circ}14'33.5''$	$B=42^{\circ}26'30.1''$ $b=5222.048$ $c=4998.842$ $A'=3868.919$	$r=1446.811$ $r_1=10169.62$ $r_2=3474.63$ $r_3=3278.239$
10. $a=44569.33$ $A=80^{\circ}32'57.54''$ $C=45^{\circ}36'2.36''$	$B=53^{\circ}51'0.1$ $b=36483.84$ $c=32282.09$ $A'=22591.29$	$r=10250.9$ $r_1=48014.57$ $r_2=28780.24$ $r_3=23821.26$
11. $a=8984.798$ $A=72^{\circ}48'48.6''$ $C=36^{\circ}36'12.72''$	$B=70^{\circ}34'58.68''$ $b=8869.832$ $c=5607.804$ $A'=4702.37$	$r=2025.334$ $r_1=8651.132$ $r_2=8303.546$ $r_3=3880.151$
12. $a=381.437$ $A=67^{\circ}10'25.8''$ $C=75^{\circ}31'14.5''$	$B=37^{\circ}18'19.7''$ $b=250.818$ $c=400.7024$ $A'=206.9235$	$r=89.677$ $r_1=342.9778$ $r_2=174.3436$ $r_3=400.0496$
13. $a=176.01$ $A=66^{\circ}19'31.38''$ $B=63^{\circ}39'57.86''$	$C=58^{\circ}0'30.76''$ $b=172.24$ $c=147.24$ $A'=96.0921$	$r=46.8754$ $r_1=161.8895$ $r_2=153.8063$ $r_3=115.5479$
14. $a=443.387$ $B=27^{\circ}28'41.02''$ $C=55^{\circ}29'1.08''$	$A=97^{\circ}2'17.90''$ $b=206.136$ $c=368.109$ $A'=223.3767$	$r=74.00344$ $r_1=575.4995$ $r_2=124.4027$ $r_3=267.6083$
15. $a=1266.82$ $A=81^{\circ}9'58.5''$ $C=33^{\circ}18'34.24''$	$B=65^{\circ}31'27.26''$ $b=1166.82$ $c=704.04$ $A'=641.0137$	$r=258.7082$ $r_1=1343.8572$ $r_2=1009.5812$ $r_3=469.3244$

EXAMPLES. V.—Continued.

DATA.	ANSWERS.
16. $\alpha=525.069$ $A=81^{\circ}17'39.16''$ $B=62^{\circ}59'59.52''$	$C=35^{\circ}42'29.32''$ $b=473.283$ $c=310.032$ $K=265.5946$ $r=110.8561$ $r_1=561.644$ $r_2=400.8712$ $r_3=210.7192$
17. $\alpha=122.073$ $A=64^{\circ}31'45.28''$ $B=65^{\circ}48'27.70''$	$C=49^{\circ}39'47.02''$ $b=123.34$ $c=103.068$ $K=67.60763$ $r=32.93385$ $r_1=109.9998$ $r_2=112.73784$ $r_3=80.62681$
18. $\alpha=786.33$ $A=21^{\circ}33'9.62''$ $C=57^{\circ}14'20.48''$	$A=101^{\circ}12'29.90''$ $b=294.48$ $c=674.11$ $K=400.8097$ $r=110.9599$ $r_1=1068.3954$ $r_2=167.0089$ $r_3=478.7944$
19. $\alpha=2358.99$ $A=80^{\circ}10'22.64''$ $B=67^{\circ}33'33.80''$	$C=26^{\circ}16'3.56''$ $b=2155.23$ $c=1046.34$ $K=1182.131$ $r=408.0798$ $r_1=2614.54$ $r_2=1068.843$ $r_3=652.2222$
20. $\alpha=70.48$ $A=28^{\circ}46'16.82''$ $B=67^{\circ}1'50.08''$	$C=84^{\circ}11'53.10''$ $b=134.822$ $c=145.682$ $K=73.21606$ $r=26.9545$ $r_1=45.0119$ $r_2=116.223$ $r_3=158.5638$
21. $\alpha=4322.56$ $A=72^{\circ}19'11.54''$ $C=51^{\circ}21'10.96''$	$B=56^{\circ}19'37.50''$ $b=3775.64$ $c=3543.32$ $K=2268.424$ $r=1094.909$ $r_1=4253.906$ $r_2=3116.297$ $r_3=2798.406$
22. $\alpha=1204.66$ $A=74^{\circ}14'44.72''$ $C=37^{\circ}20'44.36''$	$A=68^{\circ}24'30.92''$ $b=1240.9$ $c=785.92$ $K=647.7839$ $r=281.4538$ $r_1=1100.2722$ $r_2=1225.26$ $r_3=547.0578$
23. $\alpha=187.035$ $A=84^{\circ}46'54.70''$ $B=61^{\circ}58'11.62''$	$C=33^{\circ}14'53.68''$ $b=165.783$ $c=102.972$ $K=93.90667$ $r=37.29846$ $r_1=208.0306$ $r_2=136.8516$ $r_3=68.04294$
24. $\alpha=20077.67$ $A=82^{\circ}31'16.78''$ $C=32^{\circ}1'18.18''$	$B=65^{\circ}27'25.04''$ $b=18420.33$ $c=10737.33$ $K=10124.96$ $r=3982.967$ $r_1=21597.22$ $r_2=15821.55$ $r_3=7064.05$
25. $\alpha=5618.156$ $B=71^{\circ}46'44.56''$ $C=54^{\circ}46'38.52''$	$A=53^{\circ}26'36.92''$ $b=6643.4$ $c=5713.6$ $K=3497.05$ $r=1696.249$ $r_1=4524.564$ $r_2=6503.416$ $r_3=4656.465$
26. $\alpha=152.294$ $A=60^{\circ}58'15.38''$ $B=78^{\circ}51'16.72''$	$C=40^{\circ}10'27.9''$ $b=170.89$ $c=112.3631$ $K=87.08746$ $r=38.54804$ $r_1=128.2040$ $r_2=179.0552$ $r_3=79.63866$
27. $\alpha=7710.144$ $B=51^{\circ}22'32.94''$ $C=63^{\circ}34'12.98''$	$A=65^{\circ}3'14.08''$ $b=6643.4$ $c=7614.7$ $K=4251.738$ $r=2087.92$ $r_1=7004.926$ $r_2=5283.446$ $r_3=6806.502$

EXAMPLES. V.—Continued.

DATA.		ANSWERS.	
28.	$a=854.45$ $b=61^{\circ}45'17.76''$ $c=40^{\circ}45'14.78''$	$A=77^{\circ}29'27.46''$ $B=771.0144$ $C=571.36$ $R=437.6132$	$r=195.7687$ $r_1=881.4266$ $r_2=656.7996$ $r_3=407.9952$
29.	$a=1999.17$ $B=60^{\circ}46'54''$ $C=52^{\circ}6'26.74''$	$A=67^{\circ}3'39.26''$ $b=1894.38$ $c=1714.08$ $R=1085.274$	$r=533.2875$ $r_1=1857.858$ $r_2=1044.306$ $r_3=1372.217$
30.	$a=856.41$ $A=75^{\circ}13'54.42''$ $B=59^{\circ}17'26.38''$	$C=45^{\circ}28'39.2''$ $b=761.47$ $c=631.40$ $R=442.8341$	$r=206.7065$ $r_1=816.0095$ $r_2=640.079$ $r_3=471.3543$

VI.—Triangles in which three sides are given.

First find r the radius of the inscribed circle by the formula $r = \sqrt{\frac{s(s-a)(s-b)(s-c)}{s}}$; then the three half angles from the equations $\tan \frac{1}{2}A = \frac{r}{s-a}$, $\tan \frac{1}{2}B = \frac{r}{s-b}$, $\tan \frac{1}{2}C = \frac{r}{s-c}$, correcting the results so as to make the sum of the three half angles exactly 90° . In general this will not involve alterations beyond $\pm .02''$; should greater changes be required, the work must be suspected. The values of r_1, r_2, r_3 may then be obtained from the equations $r_1 = \tan \frac{1}{2}A$, $r_2 = \tan \frac{1}{2}B$, $r_3 = \tan \frac{1}{2}C$, and $R = \frac{1}{r_1}(r_1 + r_2 + r_3 - r)$. Should a further check be desired, $2R$ may be calculated from one of the three $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, $\frac{c}{\sin C}$, taking the angle nearest 45° . An example is worked out below.

DATA.		
$a=6634.39$	$s-a=1426.719$	$\log s-a=3.1543384$
$b=5646.67$	$s-b=2414.439$	$\log s-b=3.3828162$
$c=3841.158$	$s-c=4219.951$	$\log s-c=3.6253075$
$2s=16122.218$	$s=8061.109$	$-\log s=3.9063948$
$s=8061.109$		$2 \log r=6.2560673$
		$\log r=3.1280337$

$L \tan \frac{1}{2}A$	$L \tan \frac{1}{2}B$	$L \tan \frac{1}{2}C$
13.1280337	13.1280337	13.1280337
3.1543384	3.3828162	3.6253075
9.9736953	9.7452175	9.5027262
$43^{\circ}16' - \left. \begin{array}{l} 7073 \\ 2532 \end{array} \right\} \begin{array}{l} 120 \\ 120 \end{array}$	$29^{\circ}5' - \left. \begin{array}{l} 403 \\ 2975 \end{array} \right\} \begin{array}{l} 228 \\ 228 \end{array}$	$17^{\circ}39' + \left. \begin{array}{l} 6721 \\ 4371 \end{array} \right\} \begin{array}{l} 541 \\ 541 \end{array}$
7200(2.84)	13680(4.6)	32460(7.43)
2136	1780	1863
110		115

$\frac{1}{2}A=43^{\circ}15'57.16''$	$\log r=3.1280337$	$\frac{1}{2}1.9736953$	$\frac{1}{2}1.7452175$	$\frac{1}{2}1.5027262$
$\frac{1}{2}B=29^{\circ}4'55.40''$	113	3.9063948	3.9063948	3.9063948
$\frac{1}{2}C=17^{\circ}39'7.44''$	224	$\log r_1=3.8800901$	$\log r_2=3.6516123$	$\log r_3=3.4091210$
		873	075	2
$90^{\circ}0'0''$	<u>1342.87</u>	<u>7587.348</u>	<u>4483.449</u>	<u>2565.199</u>
		28	48	-2
$-r=-1342.87$	$\log r=3.5844621$	1784		
$r_1=7587.348$	$-\log \sin C=-1.7618208$	15-446	$A=86^{\circ}31'54.32''$	$r_1=7587.348$
$r_2=4483.449$	442	-12=-3	$B=58^{\circ}9'50.8''$	$r_2=4483.449$
$r_3=2565.199$	$\log 2R=3.8225971$	30	$C=35^{\circ}18'14.88''$	$r_3=2565.199$
<u>$4R=13293.126$</u>	<u>6646.562</u>	41	$r=1342.87$	$R=3323.281$

EXAMPLES. VI.

DATA.

ANSWERS.

1. $a=159928.8$ $b=108792.1$ $c=183626.3$	$A=59^{\circ}59'59.8''$ $B=36^{\circ}5'39.6''$ $C=83^{\circ}54'20.6''$ $r=28240.42$	$r_1=130581.3$ $r_2=73694.15$ $r_3=203310.9$ $R=92334.98$	
2. $a=8175.801$ $b=9387.248$ $c=5561.611$	$A=60^{\circ}$ $B=83^{\circ}54'20.5''$ $C=36^{\circ}5'39.5''$ $r=1955.214$	$r_1=6675.515$ $r_2=10393.55$ $r_3=3707.335$ $R=4720.299$	$\Delta_2=3825.637$ $=b-c$
3. $a=9943.485$ $b=8660.254$ $c=5891.161$	$A=83^{\circ}54'20.5''$ $B=60^{\circ}$ $C=36^{\circ}5'39.5''$ $r=2071.068$	$r_1=11000.415$ $r_2=7071.068$ $r_3=3990.584$ $R=5000$	$\Delta_1=2071.067$ $\Delta_2=4052.325$ $\Delta_3=2071.067$
4. $a=28466.08$ $b=24792.47$ $c=16865.15$	$A=83^{\circ}54'20.4''$ $B=60^{\circ}$ $C=36^{\circ}5'39.6''$ $r=5929.031$	$r_1=31517.57$ $r_2=20242.97$ $r_3=11424.21$ $R=14313.93$	$\Delta_1=5929.008$ $\Delta_2=11600.93$ $\Delta_3=5929.019$
5. $a=2356.186$ $b=2365.324$ $c=1383.01$	$A=72^{\circ}36'20.57''$ $B=73^{\circ}19'40.76''$ $C=34^{\circ}3'52.67''$ $r=511.3707$	$r_1=2242.342$ $r_2=2272.171$ $r_3=935.0524$ $R=1234.549$	
6. $a=38714.69$ $b=33539.37$ $c=28471.26$	$A=76^{\circ}46'57.14''$ $B=57^{\circ}29'52.58''$ $C=45^{\circ}43'10.28''$ $r=9229.178$	$r_1=39904.455$ $r_2=27628.73$ $r_3=21232.35$ $R=19884.09$	
7. $a=468.2137$ $b=331.2970$ $c=258.9733$	$A=104^{\circ}18'19.36''$ $B=43^{\circ}17'8.40''$ $C=32^{\circ}24'32.24''$ $r=78.54318$	$r_1=681.13255$ $r_2=209.9995$ $r_3=153.8038$ $R=241.5982$	
8. $a=2846.608$ $b=2479.247$ $c=1687.515$	$A=83^{\circ}53'9.77''$ $B=59^{\circ}59'46.97''$ $C=36^{\circ}7'3.26''$ $r=593.149$	$r_1=3151.134$ $r_2=2024.438$ $r_3=1143.37$ $R=1431.446$	$\Delta_1=592.3827$ $\Delta_2=592.4747$ $\Delta_3=1159.2203$

EXAMPLES. VI.—Continued.

DATA.		ANSWERS.		
9.	$a=47674.27$ $b=46639.05$ $c=33611.96$	$A=70^{\circ}46'47.08''$ $B=67^{\circ}28'53.44''$ $C=41^{\circ}44'19.48''$ $r=11571.21$	$r_1=45438.86$ $r_2=42723.55$ $r_3=24385.79$ $\hat{A}=25244.25$	$\Delta_1=7284.168$
10.	$a=466.3904$ $b=456.2141$ $c=270.8213$	$A=75^{\circ}0'0''$ $B=70^{\circ}52'57.7''$ $C=34^{\circ}7'2.3''$ $r=100$	$r_1=457.8737$ $r_2=424.7102$ $r_3=183.1016$ $\hat{A}=241.4214$	$\Delta_1=100$ $\Delta_2=159.8853$ $\Delta_3=60.32828$
11.	$a=19674.04$ $b=15361.02$ $c=9837.02$	$A=100^{\circ}21'30.58''$ $B=50^{\circ}10'45.12''$ $C=29^{\circ}27'44.3''$ $r=3312.615$	$r_1=26908.75$ $r_2=10504.845$ $r_3=5899.026$ $\hat{A}=10000$	
12.	$a=9682.458$ $b=9990.51$ $c=2919.436$	$A=75^{\circ}31'20.68''$ $B=87^{\circ}30'11.67''$ $C=16^{\circ}58'27.65''$ $r=1250$	$r_1=8749.988$ $r_2=10814.376$ $r_3=1685.044$ $\hat{A}=5000$	
13.	$a=47674.27$ $b=46639.05$ $c=33611.96$	$A=70^{\circ}46'47.08''$ $B=67^{\circ}28'53.44''$ $C=41^{\circ}44'19.48''$ $r=11571.21$	$r_1=45438.76$ $r_2=42723.55$ $r_3=24385.78$ $\hat{A}=25244.25$	
14.	$a=375.3259$ $b=312.7715$ $c=271.0686$	$A=79^{\circ}39'36.48''$ $B=55^{\circ}3'53.24''$ $C=45^{\circ}16'30.28''$ $r=86.95647$	$r_1=400$ $r_2=250$ $r_3=200$ $\hat{A}=190.7608$	
15.	$a=14538.87$ $b=34045.37$ $c=30139.84$	$A=24^{\circ}42'1.5''$ $B=95^{\circ}16'15.9''$ $C=60^{\circ}1'42.6''$ $r=5500.802$	$r_1=8684.138$ $r_2=43489.62$ $r_3=22712.04$ $\hat{A}=17396.25$	
16.	$a=7406.25$ $b=31560.45$ $c=30137.34$	$A=13^{\circ}32'3.74''$ $B=94^{\circ}13'58.62''$ $C=72^{\circ}13'57.64''$ $r=3221.169$	$r_1=4100.008$ $r_2=37203.835$ $r_3=25210.84$ $\hat{A}=15823.38$	
17.	$a=10927.86$ $b=8517.08$ $c=3642.62$	$A=122^{\circ}47'45.58''$ $B=40^{\circ}55'54.4''$ $C=16^{\circ}16'20.02''$ $r=1129.583$	$r_1=21171.03$ $r_2=4308.21$ $r_3=1650.345$ $\hat{A}=6500$	
18.	$a=5761.367$ $b=5017.848$ $c=3413.403$	$A=83^{\circ}54'20.57''$ $B=60^{\circ}0'0''$ $C=36^{\circ}5'39.43''$ $r=1200$	$r_1=6378.981$ $r_2=4097.055$ $r_3=2312.188$ $\hat{A}=2897.056$	$\Delta_1=\Delta_2=r$ $\Delta_3=2347.964$ $=a-c$
19.	$a=94371.5$ $b=124351.6$ $c=106381.7$	$A=47^{\circ}30'$ $B=76^{\circ}17'14.52''$ $C=56^{\circ}12'45.48''$ $r=30000.31$	$r_1=71524.76$ $r_2=127657.63$ $r_3=86817.93$ $\hat{A}=64000$	$\Delta_1=15998.76$
20.	$a=4157.855$ $b=4812.793$ $c=3608.528$	$A=57^{\circ}2'28''$ $B=76^{\circ}13'21.6''$ $C=46^{\circ}44'10.4''$ $r=1158.427$	$r_1=3417.89$ $r_2=4933.676$ $r_3=2717.593$ $\hat{A}=2477.683$	

EXAMPLES. VI.--Continued.

DATA.		ANSWERS.			
21.	$a=9401.45$ $b=7360.12$ $c=5318.79$	The sides in A, P , and the greatest angle exceeds the least by 60° . $8r=3A$.	$A=94^\circ 20' 27.92''$ $B=51^\circ 19' 4.16''$ $C=34^\circ 20' 27.92''$ $r=1767.844$	$r_1=11910.02$ $r_2=5303.53$ $r_3=3411.289$ $A=4714.249$	$\Delta_1=2357.125$
22.	$a=2692538$ $b=1860498$ $c=1028458$		$A=135^\circ 31' 20.04''$ $B=28^\circ 57' 18.08''$ $C=15^\circ 31' 20.98''$ $r=240189.2$	$r_1=6125317$ $r_2=720507.9$ $r_3=380361.9$ $A=1921514$	$\Delta_1=1664081$
23.	$a=964.701$ $b=955.078$ $c=945.455$	The sides in A, P , $A-C=2^\circ$.	$A=61^\circ 0' 18.14''$ $B=59^\circ 59' 23.70''$ $C=59^\circ 0' 18.16''$ $r=275.6513$	$r_1=843.9606$ $r_2=820.9536$ $r_3=810.6189$ $A=551.4705$	$\Delta_1=9.622469$
24.	$a=2089.88$ $b=2413.68$ $c=1206.96$		$A=59^\circ 58' 46.83''$ $B=90^\circ 1' 1.35''$ $C=30^\circ 0' 11.82''$ $r=441.7114$	$r_1=1647.81$ $r_2=2856.109$ $r_3=765.1524$ $A=1206.84$	$\Delta_1=624.7486$ $\Delta_2=1208.062$ $\Delta_3=625.855$
25.	$a=4794.217$ $b=4329.393$ $c=3655.232$		$A=73^\circ 15' 10.41''$ $B=59^\circ 51' 12.13''$ $C=46^\circ 53' 37.46''$ $r=1185.846$	$r_1=4749.779$ $r_2=3078.041$ $r_3=2771.157$ $A=2503.283$	
26.	$a=544.36$ $b=426.39$ $c=425.09$		$A=79^\circ 28' 50.42''$ $B=50^\circ 21' 53.56''$ $C=50^\circ 9' 16.02''$ $r=127.6707$	$r_1=580.2548$ $r_2=328.155$ $r_3=326.5914$ $A=276.8326$	
27.	$a=51509.95$ $b=72027.91$ $c=63926$		$A=44^\circ 1' 38.62''$ $B=76^\circ 22' 16.08''$ $C=59^\circ 36' 5.3''$ $r=17070.53$	$r_1=37896.22$ $r_2=73721.52$ $r_3=53682.39$ $A=37057.2$	
28.	$a=10301.99$ $b=10825.782$ $c=9205.4$		$A=61^\circ 16' 11.50''$ $B=67^\circ 8' 35.18''$ $C=51^\circ 35' 13.32''$ $r=2880.918$	$r_1=8981.956$ $r_2=10065.804$ $r_3=7329.696$ $A=5874.135$	
29.	$a=5844.305$ $b=7202.791$ $c=4602.7$		$A=54^\circ 4' 4.86''$ $B=86^\circ 18' 43.14''$ $C=39^\circ 37' 12''$ $r=1520.916$	$r_1=4503.11$ $r_2=8274.385$ $r_3=3178.898$ $A=3608.869$	
30.	$a=1168.861$ $b=1082.5782$ $c=1278.52$		$A=58^\circ 38' 46.14''$ $B=52^\circ 16' 21.68''$ $C=69^\circ 4' 52.18''$ $r=334.8419$	$r_1=991.3952$ $r_2=806.0432$ $r_3=1214.8782$ $A=684.3687$	
31.	$a=1262.92$ $b=1332.60$ $c=1142.72$		$A=60^\circ 46' 54.01''$ $B=67^\circ 3' 39.26''$ $C=52^\circ 9' 26.73''$ $r=355.525$	$r_1=1096.2042$ $r_2=1238.5722$ $r_3=914.8112$ $A=723.5157$	
32.	$a=18943.8$ $b=25692.3$ $c=22844.1$		$A=45^\circ 28' 39.20''$ $B=75^\circ 13' 54.42''$ $C=59^\circ 17' 26.38''$ $r=6201.195$	$r_1=14140.63$ $r_2=25998.28$ $r_3=19202.37$ $A=13285.02$	

EXAMPLES. VI.--Continued.

DATA.		ANSWERS.	
33.	$a=70137$ $b=66630$ $c=76147$	$A=58^{\circ}23'20.49''$ $B=54^{\circ}09.25'$ $C=71^{\circ}36'30.26''$ $r=20293.99$	$r_1=59483.42$ $r_2=54245.50$ $r_3=71278.06$ $K=41178.20$
34.	$a=3506.8$ $b=4282.05$ $c=2856.8$	$A=54^{\circ}31'24.61''$ $B=83^{\circ}55'0.40''$ $C=41^{\circ}33'34.99''$ $r=935.773$	$r_1=2742.83$ $r_2=4785.72$ $r_3=2019.82$ $K=2153.15$
35.	$a=786.934$ $b=604.81$ $c=431.5957$	$A=97^{\circ}23'42.2''$ $B=49^{\circ}39'21.88''$ $C=32^{\circ}56'55.92''$ $r=141.9714$	$r_1=1037.641$ $r_2=421.7921$ $r_3=269.6064$ $K=396.767$
36.	$a=786.934$ $b=748.887$ $c=431.5957$	$A=78^{\circ}35'37.44''$ $B=68^{\circ}53'4.20''$ $C=32^{\circ}31'18.36''$ $r=161.04$	$r_1=805.0664$ $r_2=674.625$ $r_3=286.9277$ $K=401.3948$
37.	$a=43783$ $b=43397$ $c=43012$	$A=60^{\circ}53'9.3''$ $B=59^{\circ}59'29.07''$ $C=50^{\circ}7'21.63''$ $r=12525.76$	$r_1=38257.23$ $r_2=37576.69$ $r_3=36921.59$ $K=25057.44$ $\Delta_1=385.1494$ $\Delta_2=771.168$ $\Delta_3=399.6372$
38.	$a=96.264$ $b=114.735$ $c=106.848$	$A=51^{\circ}20'53.50''$ $B=68^{\circ}33'38.10''$ $C=60^{\circ}5'28.34''$ $r=30.12111$	$r_1=76.39623$ $r_2=108.3302$ $r_3=91.92327$ $K=61.63215$ $\Delta_1=9.255574$
39.	$a=224.626$ $b=218.323$ $c=199.92$	$A=64^{\circ}47'27.34''$ $B=61^{\circ}34'17.14''$ $C=53^{\circ}38'15.52''$ $r=61.428976$	$r_1=203.9497$ $r_2=191.5018$ $r_3=162.4983$ $K=124.1302$
40.	$a=14724$ $b=27601$ $c=17224$	$A=50^{\circ}0'30.76''$ $B=66^{\circ}10'31.38''$ $C=63^{\circ}39'57.86''$ $r=4687.54$	$r_1=11554.79$ $r_2=16188.95$ $r_3=15380.63$ $K=9609.21$
41.	$a=63341$ $b=52587$ $c=29448$	$A=97^{\circ}2'17.00''$ $B=55^{\circ}29'1.08''$ $C=27^{\circ}28'41.02''$ $r=10571.92$	$r_1=82214.21$ $r_2=38229.75$ $r_3=17771.81$ $K=31910.96$
42.	$a=1266.82$ $b=704.04$ $c=1166.82$	$A=81^{\circ}9'58.50''$ $B=33^{\circ}18'34.24''$ $C=65^{\circ}31'27.26''$ $r=258.7082$	$r_1=1343.8572$ $r_2=469.3244$ $r_3=1009.5812$ $K=641.0137$
43.	$a=1750.23$ $b=1577.61$ $c=1033.44$	$A=81^{\circ}17'39.16''$ $B=62^{\circ}59'51.52''$ $C=35^{\circ}42'29.32''$ $r=399.5202$	$r_1=1872.1467$ $r_2=336.2372$ $r_3=702.3975$ $K=885.3153$
44.	$a=13467.87$ $b=8287.92$ $c=7251.93$	$A=120^{\circ}$ $B=32^{\circ}12'15.22''$ $C=27^{\circ}47'44.78''$ $r=1794.387$	$r_1=25121.42$ $r_2=4186.903$ $r_3=3588.774$ $K=7775.68$ $\Delta_2=15539.85$ $=b+c$

VII.—Triangles in which two sides and the included angle are given.

Let b, c be the two given lengths ($b > c$), A the included angle, then we find $\frac{1}{2}(B - C)$ by the equation $\tan \frac{1}{2}(B - C) = \frac{b - c}{b + c} \cot \frac{1}{2}A$; and $\frac{1}{2}(B + C) = 90^\circ - \frac{A}{2}$; $B = \frac{1}{2}(B + C) + \frac{1}{2}(B - C)$, $C = \frac{1}{2}(B + C) - \frac{1}{2}(B - C)$. The value of $2R$ is then calculated from each of the expressions $b/\sin B$, $c/\sin C$, which should agree ($p.p.$). The formulae $a = 2R \sin A$, $r = (s - a) \tan \frac{A}{2}$, $r_1 = s \tan \frac{A}{2}$, $r_2 = (s - c) \cot \frac{1}{2}A$, $r_3 = (s - b) \cot \frac{1}{2}A$ enable us to find a, r, r_1, r_2, r_3 ; and the equation $4R = r_1 + r_2 + r_3 - r$ supplies another check.

DATA.		EXAMPLE WORKED OUT.	
$b = 334.17$	$b - c = 99.90$	$I. \cot \frac{1}{2}A = 10.5228579$	4592
$c = 234.27$	$b + c = 568.44$	$\log(b - c) = 3.9995655$	$10^\circ = 765 \quad 3$
$A = 33^\circ 23' 37''$	$A = 16^\circ 41' 48.5''$	$-\log(b + c) = 4.7546846$	$1.5 = 114 \quad 8$
		$I \tan \frac{1}{2}(B - C) = 9.7678268$	4560(1.58
		$30^\circ 22' -$	344
		$2896 \quad 76$	1664
			216
$\frac{1}{2}(B - C) = 30^\circ 21' 58.42''$		$\log b = 4.5239675$	$308 = 5 + 16 + 16$
$\frac{1}{2}(B + C) = 73^\circ 18' 11.5''$		$-\log \sin B = -1.9874955$	257
$B = 103^\circ 40' 9.92''$	$180^\circ - B = 76^\circ 19' 50.08''$	$\log 2R = 4.5364463$	250 4
$C = 42^\circ 56' 13.08''$			5 7
			1 7
		34391.12	15
$\log c = 4.3697167$	1358	$\log 2R = 4.5364463$	1916
$-\log \sin C = 1.8332408$	$13 - 271 \quad 6$	$\log \sin A = 1.7405505$	36 = 1149 6
	$1.2 = 27 \quad 2$		1 = 31 9
$\log 2R = 4.5364463$	$1.12 = 2 \quad 7$	$\log a = 4.2771149$	
		947	
		18928.44	102
$\log \cot \frac{1}{2}A = 0.5229459$	$a = 18928.44$	$\log(s - a) = 4.2777877$	
$\log \tan \frac{1}{2}A = 1.4770541$	$b = 334.17$	$\log(r/b) = 3.6502317$	
	$c = 234.27$	$\log(s - c) = 4.1601449$	
	75772.44	$\log r = 4.5784813$	
	$s = 37886.22$		
$\log r$	$\log r_1$	$\log r_2$	$\log r_3$
1.4770541	1.4770541	.5229459	.5229459
4.2777877	4.5784813	4.1601449	3.6502317
3.7548418	4.0555354	4.6830908	4.1731776
374	12	831	571
5686.457	11364.11	48204.86	11899.70
44	42	77	250

ANSWERS.

$B = 103^\circ 40' 9.92''$	$a = 18928.44$	$r = 5686.457$	$r_1 = 48204.86$
$C = 42^\circ 56' 13.08''$	$R = 17195.56$	$r_2 = 11364.11$	$r_3 = 14899.70$

EXAMPLES. VII.

DATA.

ANSWERS.

1. $b=23571.26$ $c=20753.82$ $A=67^{\circ}31'55''$		$B=61^{\circ}39'53.64''$ $C=50^{\circ}48'11.36''$ $a=24747.05$ $A=13389.91$	$r=6544.85$ $r_1=23090.21$ $r_2=20614.16$ $r_3=16400.11$	
2. $b=8438.715$ $c=4219.357$ $A=59^{\circ}37'56.4''$		$B=90^{\circ}22'6.07''$ $C=29^{\circ}59'57.53''$ $a=7281.067$ $R=4219.444$	$r=1540.727$ $r_1=5713.363$ $r_2=10033.872$ $r_3=2671.274$	
3. $b=378.9217$ $c=269.1351$ $A=88^{\circ}59'2.4''$		$B=55^{\circ}17'30.72''$ $C=35^{\circ}43'26.88''$ $a=460.8674$ $R=230.4699$	$r=91.94954$ $r_1=544.71625$ $r_2=200.4319$ $r_3=178.6812$	
4. $b=13347.61$ $c=11258.73$ $A=47^{\circ}31'57''$		$B=77^{\circ}8'44''$ $C=55^{\circ}19'19''$ $a=10099.1$ $R=6845.362$	$r=3194.125$ $r_1=7641.255$ $r_2=13839$ $r_3=9095.308$	
5. $b=99144.49$ $c=60876.14$ $A=45''$		$B=97^{\circ}30'$ $C=37^{\circ}30'$ $a=70710.68$ $R=50000$	$r=18496.69$ $r_1=47786.01$ $r_2=131549.34$ $r_3=39161.36$	
6. $b=5.6794$ $c=5.6318$ $A=44^{\circ}59'6.26''$		$B=68^{\circ}5'23.27''$ $C=66^{\circ}55'30.47''$ $a=4.327479$ $R=3.060782$	$r=1.445844$ $r_1=3.237680$ $r_2=5.283127$ $r_3=5.168167$	
7. $b=87.92154$ $c=29.30718$ $A=70^{\circ}31'43.62''$		$B=90'$ $C=19^{\circ}28'16.38''$ $a=82.89322$ $R=43.96077$	$r=12.13143$ $r_1=70.75379$ $r_2=100.06097$ $r_3=17.16775$	
8. $b=99905.1$ $c=29194.36$ $A=75^{\circ}31'20.7''$		$B=87^{\circ}30'11.65''$ $C=16^{\circ}58'27.65''$ $a=96824.54$ $R=50000$	$r=12500$ $r_1=87499.9$ $r_2=108143.67$ $r_3=16856.4$	
9. $b=1243.516$ $c=1003.817$ $A=47^{\circ}30'$		$B=76^{\circ}17'14.56''$ $C=56^{\circ}12'45.44''$ $a=943.715$ $R=640$	$r=300.0031$ $r_1=715.2478$ $r_2=1276.5762$ $r_3=868.1792$	$\Delta_1=159.9976$ $\Delta_2=343.1666$ $\Delta_3=185.617$
10. $b=57777.78$ $c=47222.22$ $A=129^{\circ}18'27.52''$	$180r=9r_1=76r_2=95r_3$	$B=28^{\circ}4'21.01''$ $C=22^{\circ}37'11.47''$ $a=95000$ $R=61388.89$	$r=10555.56$ $r_1=211111.1$ $r_2=25000$ $r_3=20000$	
11. $b=5237.828$ $c=6673.472$ $A=87^{\circ}50'57.8''$		$B=38^{\circ}56'32.8''$ $C=53^{\circ}12'29.4''$ $a=8327.466$ $R=4166.667$	$r=1725.888$ $r_1=9746.503$ $r_2=3577.745$ $r_3=5068.313$	$\Delta_1=1725.894$
12. $b=14045.86$ $c=8109.38$ $A=30^{\circ}$		$B=120^{\circ}$ $C=30^{\circ}$ $a=8109.38$ $R=8109.38$	$r=1881.79$ $r_1=4054.69$ $r_2=26209.93$ $r_3=4054.69$	

EXAMPLES. VII.—Continued.

DATA.	ANSWERS.
13. $b=1404.59$ $c=810.94$ $A=30^{\circ}0'0.07''$	$B=119^{\circ}59'59.965''$ $C=29^{\circ}59'59.965''$ $a=810.941$ $R=810.9405$ $r=188.1794$ $r_1=405.4705$ $r_2=2621.0007$ $r_3=405.4702$
14. $b=4568.39$ $c=864.29$ $A=110^{\circ}34'11.96''$	$B=60^{\circ}$ $C=9^{\circ}25'48.04''$ $a=4938.8$ $R=2637.561$ $r=356.427$ $r_1=7484.971$ $r_2=2993.989$ $r_3=427.712$ $\Delta_1=2253.115$ $\Delta_2=4074.51$ $\Delta_3=2253.115$
15. $b=9237.97$ $c=6598.55$ $A=81^{\circ}47'12.46''$	$B=60^{\circ}$ $C=38^{\circ}12'47.54''$ $a=10557.68$ $R=5333.545$ $r=2285.805$ $r_1=11429.025$ $r_2=7614.35$ $r_3=4571.61$ $\Delta_1=2015.891$ $\Delta_2=3959.131$ $\Delta_3=2015.891$
16. $b=47898.32$ $c=39817.65$ $A=65^{\circ}26'23.6''$	$B=65^{\circ}26'23.6''$ $C=49^{\circ}7'12.8''$ $a=47898.32$ $R=26331.48$ $r=12791.04$ $r_1=43564.76$ $r_2=43564.76$ $r_3=30987.44$
17. $b=48127.93$ $c=36085.28$ $A=57^{\circ}2'28''$	$B=76^{\circ}13'21.62''$ $C=46^{\circ}44'10.38''$ $a=41578.55$ $R=24776.83$ $r=11584.27$ $r_1=34178.9$ $r_2=49336.76$ $r_3=27175.93$
18. $b=4157.855$ $c=3608.528$ $A=76^{\circ}13'21.6''$	$B=57^{\circ}2'28''$ $C=46^{\circ}44'10.4''$ $a=4812.793$ $R=2477.683$ $r=1158.427$ $r_1=4933.676$ $r_2=3417.89$ $r_3=2717.593$
19. $b=23571.26$ $c=20753.52$ $A=67^{\circ}31'55''$	$B=61^{\circ}39'55.86''$ $C=50^{\circ}48'9.14''$ $a=24746.85$ $R=13389.84$ $r=6544.721$ $r_1=23090.03$ $r_2=20614.22$ $r_3=16399.73$
20. $b=30901.7$ $c=58778.53$ $A=18^{\circ}$	$B=18^{\circ}$ $C=144^{\circ}$ $a=30901.7$ $R=50000$ $r=4654.803$ $r_1=9549.152$ $r_2=9549.152$ $r_3=185556.5$
21. $b=12369.12$ $c=4123.04$ $A=40^{\circ}55'55''$	The triangle in which the greatest side is 3 times the least, and the greatest angle 3 times the mean. <i>q.p.</i> $B=122^{\circ}47'44.84''$ $C=16^{\circ}16'20.16''$ $a=9640.399$ $R=7357.262$ $r=1278.566$ $r_1=4876.439$ $r_2=23963.16$ $r_3=1868.021$ $\Delta_1=5942.711$ $\Delta_2=14974.37$ $\Delta_3=9397.907$
22. $b=13571$ $c=8965$ $A=79^{\circ}55'58.5''$	$B=63^{\circ}44'18.57''$ $C=36^{\circ}19'42.93''$ $a=14899.98$ $R=7566.486$ $r=3199.883$ $r_1=15687.6$ $r_2=11636.99$ $r_3=6141.241$ $\Delta_1=2971.191$ $\Delta_2=6021.614$ $\Delta_3=3159.603$
23. $b=18791.69$ $c=11275.05$ $A=68^{\circ}35'57.6''$	$B=75^{\circ}49'39.51''$ $C=35^{\circ}34'22.89''$ $a=18045.27$ $R=9690.8$ $r=4100.197$ $r_1=16409.691$ $r_2=18736.413$ $r_3=7717.292$ $\Delta_1=3800.416$
24. $b=133.7127$ $c=206.3273$ $A=58^{\circ}21'37.4''$	$B=39^{\circ}53'33.16''$ $C=81^{\circ}44'49.44''$ $a=177.4979$ $R=104.2432$ $r=45.384$ $r_1=144.5037$ $r_2=93.9095$ $r_3=223.9436$

EXAMPLES. VII.—Continued.

DATA.		ANSWERS.	
25.	$b=8233.59$ $c=6618.853$ $A=83^{\circ}8'1.8''$	$B=55^{\circ}25'20.65''$ $C=41^{\circ}26'37.55''$ $a=9928.291$ $R=5000.006$	$r=2183.392$ $r_1=10987.9$ $r_2=6508.179$ $r_3=4687.339$
26.	$b=23870.56$ $c=10775.56$ $A=41^{\circ}24'34.6''$	$b+c=2a, B-C=90^{\circ}$ $R=4r, r_1=3r$ $B=114^{\circ}17'42.7''$ $C=24^{\circ}17'42.7''$ $a=17323.06$ $R=13095$	$r=3273.749$ $r_1=9821.247$ $r_2=40239.337$ $r_3=5593.201$
27.	$b=5361.796$ $c=5957.551$ $A=23^{\circ}33'23.3''$	$B=64^{\circ}3'19.87''$ $C=92^{\circ}23'10.83''$ $a=2383.022$ $R=2981.364$	$r=931.6764$ $r_1=1428.571$ $r_2=4285.715$ $r_3=7142.852$
28.	$b=138.74$ $c=116.82$ $A=34^{\circ}33'37''$	$B=88^{\circ}8'4.07''$ $C=57^{\circ}18'18.93''$ $a=78.7452$ $R=69.4068$	$r=27.50223$ $r_1=51.99869$ $r_2=161.7969$ $r_3=91.33386$
29.	$b=1687.48$ $c=2531.22$ $A=104^{\circ}28'39.04''$	$B=28^{\circ}57'18.08''$ $C=46^{\circ}34'2.88''$ $a=3374.96$ $R=1742.822$	$r=544.6318$ $r_1=4901.688$ $r_2=980.3372$ $r_3=1633.896$
30.	$b=31428.54$ $c=35714.25$ $A=69^{\circ}47'25.64''$	$74r=20r_1=30r_2=24r_3$ $B=49^{\circ}52'33.45''$ $C=60^{\circ}20'0.91''$ $a=38571.41$ $R=20550.91$	$r=9964.067$ $r_1=36867.08$ $r_2=24578.06$ $r_3=30722.57$
31.	$b=273.8163$ $c=262.1835$ $A=8^{\circ}13'57.6''$	$B=102^{\circ}39'55.14''$ $C=69^{\circ}6'7.26''$ $a=40.18648$ $R=140.323$	$r=17.8412$ $r_1=20.73332$ $r_2=360.02$ $r_3=198.3801$
32.	$b=24.1368$ $c=12.0696$ $A=59^{\circ}58'48''$	$B=90^{\circ}1'0.17''$ $C=30^{\circ}0'11.83''$ $a=20.89888$ $R=12.068405$	$r=4.417119$ $r_1=16.47823$ $r_2=28.56097$ $r_3=7.651542$
33.	$b=856.41$ $c=761.47$ $A=45^{\circ}28'39.2''$	$B=75^{\circ}13'54.53''$ $C=59^{\circ}17'26.27''$ $a=631.4601$ $R=442.834$	$r=206.7065$ $r_1=471.3542$ $r_2=866.6096$ $r_3=640.079$
34.	$b=786.934$ $c=748.887$ $A=32^{\circ}31'18.36''$	$B=78^{\circ}35'37.44''$ $C=68^{\circ}53'4.20''$ $a=431.5957$ $R=401.3948$	$r=161.04$ $r_1=286.9277$ $r_2=805.0664$ $r_3=674.6250$
35.	$b=3734.67$ $c=3593.19$ $A=52^{\circ}31'49.2''$	$B=65^{\circ}58'31.37''$ $C=61^{\circ}29'39.43''$ $a=3245.25$ $R=2044.444$	$r=1007.332$ $r_1=2608.78$ $r_2=3431.515$ $r_3=3144.813$
36.	$b=487.139$ $c=479.363$ $A=48^{\circ}39'39.6''$	$B=66^{\circ}41'20''$ $C=64^{\circ}36'0.4''$ $a=398.2614$ $R=265.2195$	$r=128.4674$ $r_1=308.5446$ $r_2=448.9992$ $r_3=431.8017$

$\Delta_1=1067.256$
 $\Delta_2=2586.858$
 $\Delta_3=1555.78$

$\Delta_1=6.247486$
 $\Delta_2=12.08062$
 $\Delta_3=6.25855$

$\Delta_1=246.7459$
 $\Delta_2=489.2463$
 $\Delta_3=243.3927$

$\Delta_1=46.87498$
 $\Delta_2=85.8304$
 $\Delta_3=38.99333$

EXAMPLES. VII.—Continued.

DATA.		ANSWERS.		
37.	$b=433.97$ $c=430.12$ $A=60^{\circ}53'9.3''$	$B=59^{\circ}59'29.06''$ $C=59^{\circ}7'21.64''$ $a=437.83$ $R=250.5744$	$r=125.2576$ $r_1=382.5723$ $r_2=375.7669$ $r_3=369.2159$	$\Delta_1=3.851494$ $\Delta_2=3.996372$ $\Delta_3=7.71168$
38.	$b=1057.81$ $c=1220.55$ $A=27^{\circ}47'44.76''$	$B=60^{\circ}0'0''$ $C=92^{\circ}12'15.24''$ $a=569.59$ $R=610.7271$	$r=211.4054$ $r_1=352.3423$ $r_2=822.1324$ $r_3=1479.839$	$\Delta_1=338.7705$ $\Delta_2=650.9600$ $=c-a$ $\Delta_3=338.7705$

VIII.—Triangles in which two sides and the angle opposite the greater are given.

Let a, b be the two given lengths ($a > b$), and A the given angle. First calculate R by the equation $2R = a/\sin A$; B must be an acute angle and is found from $\sin B = b/2R$; C from $A + B + C = 180^{\circ}$; c from $c = 2R \sin C$. Calculate r, r_1, r_2, r_3 exactly as in V.

EXAMPLE WORKED OUT.

DATA.	$\log a = 3.6823972$ $-\log \sin A = 1.9873103$ $\log 2R = 3.6950757$ 4955.366 57	310 18 93 3.6=18 6 101 6 46°44' + } 1188 }	$\log b = 3.5573301$ $-\log 2R = 3.6950757$ $L \sin B = 9.8622544$ 338 206	12360(10.4" 480
$a=4812.793$ $b=3608.528$ $A=76^{\circ}13'21.6''$	$\log 2R = 3.6950757$ $\log \sin C = 1.9237554$ $\log c = 3.6188693$ 4157.855 57	819 24=327 6 4- 54 6	$A = 38^{\circ}6'40.8''$ $\log \tan \frac{A}{2} = 1.8943715$ 1769 $= 1.8945484$ $\log \cot \frac{A}{2} = 0.1054516$	2602 36=1561 2 4.8= 208 2
$a=4812.793$ $b=3608.528$ $c=4157.855$ $2r=12579.176$ $r=6289.588$	$s-a=1476.795$ $s-b=2681.06$ $s-c=2131.733$ $s=6289.588$	$\log -3.1693202$ $\log -3.4283065$ $\log -3.3287329$ $\log -3.7986222$		
$\log r$ 3.1693202 1.8945484 3.0638686 585 1158.427 101	$\log r_1$ 3.7986222 1.8945484 3.6931706 639 4933.676 67	$\log r_2$ 3.3287329 $.1054516$ 3.4341845 696 2717.593 149	$\log r_3$ 3.4283065 $.1054516$ 3.5337581 466 3417.89 115	$-r = -1158.427$ $r_1 = 4933.676$ $r_2 = 2717.593$ $r_3 = 3417.89$ $4R = 9910.732$

ANSWERS.

$R=2477.683$	$C=57^{\circ}2'28''$	$r=1158.427$	$r_2=2717.593$
$B=46^{\circ}44'10.4''$	$c=4157.855$	$r_1=4933.676$	$r_3=3417.89$

EXAMPLES VIII.

DATA.		ANSWERS.	
1.	$a=266.0007$ $b=138.7214$ $A=62^{\circ}27'27.4''$	$R=150$ $B=27^{\circ}32'32.6''$ $C=90^{\circ}$ $c=300$	$r=52.36105$ $r_1=213.03965$ $r_2=86.36035$ $r_3=352.36105$ $\Delta_1=82.41167$
2.	$a=2620.87$ $b=1513.16$ $A=120^{\circ}$	$R=1513.16$ $B=30^{\circ}$ $C=30^{\circ}$ $c=1513.16$	$r=351.13$ $r_1=4890.611$ $r_2=756.5801$ $r_3=756.5801$ $\Delta_1=1107.84$
3.	$a=82893.16$ $b=29307.18$ $A=70^{\circ}31'43.55''$	$R=43960.735$ $B=19^{\circ}28'16.45''$ $C=90^{\circ}$ $c=87921.47$	$r=12139.435$ $r_1=70753.725$ $r_2=17167.745$ $r_3=100060.905$ $\Delta_1=29414.78$
4.	$a=2731.843$ $b=1600.277$ $A=72^{\circ}58'7.5''$	$R=14285.715$ $B=34^{\circ}3'45''$ $C=72^{\circ}58'7.5''$ $c=2731.843$	$r=591.7336$ $r_1=2612.039$ $r_2=1081.942$ $r_3=2612.039$ $\Delta_1=591.7373$
5.	$a=27442.1$ $b=11366.89$ $A=67^{\circ}30'$	$R=14851.56$ $B=22^{\circ}30'$ $C=90^{\circ}$ $c=29703.12$	$r=4552.935$ $r_1=22889.165$ $r_2=6813.955$ $r_3=34250.055$ $\Delta_1=11629.12$
6.	$a=968.2454$ $b=291.9436$ $A=75^{\circ}31'20.68''$	$R=500$ $B=16^{\circ}58'27.68''$ $C=87^{\circ}30'11.64''$ $c=999.05114$	$r=125.0001$ $r_1=874.9987$ $r_2=168.564$ $r_3=1081.4372$ $\Delta_1=353.5534$
7.	$a=24150.16$ $b=22202.21$ $A=57^{\circ}41'57.5''$	$R=14285.72$ $B=50^{\circ}50'37.48''$ $C=71^{\circ}18'25.01''$ $c=27064.26$	$r=6918.089$ $r_1=20222.015$ $r_2=17506.506$ $r_3=26332.425$ $\Delta_1=2534.167$
8.	$a=8571.427$ $b=7142.856$ $A=73^{\circ}44'23.3''$	$R=1464.286$ $B=53^{\circ}7'18.35''$ $C=53^{\circ}7'48.35''$ $c=7142.856$	$r=2412.857$ $r_1=8571.427$ $r_2=5714.284$ $r_3=5714.284$ $\Delta_1=892.8586$
9.	$a=673.1282$ $b=336.5642$ $A=92^{\circ}27'53.4''$	$R=336.8757$ $B=29^{\circ}53'36.39''$ $C=57^{\circ}38'30.21''$ $c=569.13$	$r=121.3962$ $r_1=824.1232$ $r_2=211.003$ $r_3=433.7731$
10.	$a=47124.88$ $b=35623.91$ $A=65^{\circ}37'48.6''$	$R=25867.18$ $B=43^{\circ}31'7.76''$ $C=70^{\circ}51'3.64''$ $c=48871.84$	$r=12048.9$ $r_1=42436.38$ $r_2=26268.18$ $r_3=46813.06$
11.	$a=9401.45$ $b=7360.12$ $A=94^{\circ}20'27.9''$	$a+c=2b$ $A-C=60^{\circ}$ $R=4714.249$ $B=51^{\circ}19'4.12''$ $C=34^{\circ}20'27.98''$ $c=5318.792$	$r=1767.845$ $r_1=11910.022$ $r_2=5303.53$ $r_3=3411.291$ $\Delta_1=2357.121$
12.	$a=7791.368$ $b=5387.123$ $A=69^{\circ}31'48.12''$	$R=4158.254$ $B=40^{\circ}12'23.22''$ $C=70^{\circ}15'48.66''$ $c=7827.959$	$r=1882.335$ $r_1=7290.425$ $r_2=3854.223$ $r_3=7370.705$

EXAMPLES VIII.—Continued.

DATA.		ANSWERS.	
13.	$a=45217.25$ $b=22608.62$ $A=45^{\circ}40'24.6''$	$R=31604.12$ $B=19^{\circ}40'52.88''$ $C=114^{\circ}38'42.52''$ $c=57450.47$	$r=8087.68$ $r_1=25676.23$ $r_2=11174.34$ $r_3=97653.6$
14.	$a=967.8191$ $b=559.4229$ $A=75^{\circ}25'30''$	$R=500$ $B=34^{\circ}0'58.6''$ $C=70^{\circ}33'31.4''$ $c=942.983$	$r=206.6802$ $r_1=955.0335$ $r_2=377.801$ $r_3=873.846$
15.	$a=8045.638$ $b=4257.129$ $A=82^{\circ}37'42.62''$	$R=4056.343$ $B=31^{\circ}39'5.01''$ $C=65^{\circ}43'12.37''$ $c=7395.099$	$r=1585.028$ $r_1=8656.84$ $r_2=2791.74$ $r_3=6361.823$
16.	$a=2428.571$ $b=2142.857$ $A=58^{\circ}59'50.2''$	$R=1416.667$ $B=50^{\circ}42'25.9''$ $C=70^{\circ}17'43.9''$ $c=2667.425$	$r=670.2642$ $r_1=2097.227$ $r_2=1691.417$ $r_3=2548.286$
17.	$a=576.1367$ $b=501.7848$ $A=83^{\circ}54'20.57''$	$R=289.7056$ $B=60^{\circ}0'0''$ $C=36^{\circ}5'39.43''$ $c=341.3403$	$r=120$ $r_1=637.898$ $r_2=409.7056$ $r_3=231.2187$
18.	$a=241.368$ $b=120.696$ $A=90^{\circ}1'1.35''$	$R=120.684$ $B=30^{\circ}0'11.88''$ $C=59^{\circ}58'46.77''$ $c=208.988$	$r=44.17114$ $r_1=285.6109$ $r_2=76.51524$ $r_3=164.781$
19.	$a=4562.141$ $b=2708.213$ $A=70^{\circ}52'57.7''$	$K^2 - 2Kr = r^2, q.p.$ $\Delta_1 = r$ $R=2414.214$ $B=34^{\circ}7'2.3''$ $C=75^{\circ}0'0''$ $c=4663.904$	$r=1000$ $r_1=4247.10$ $r_2=1831.016$ $r_3=4578.739$
20.	$a=2157.87$ $b=2132.19$ $A=64^{\circ}46'37.2''$	$R=11.92647$ $B=63^{\circ}21'58.4''$ $C=51^{\circ}51'24.4''$ $c=1875.96$	$r=586.8513$ $r_1=1955.67$ $r_2=1902.85$ $r_3=1498.918$
21.	$a=4891.32$ $b=4377.24$ $A=63^{\circ}56'43.96''$	$R=2722.309$ $B=53^{\circ}30'34.76''$ $C=62^{\circ}32'41.28''$ $c=4831.4$	$r=1347.455$ $r_1=4400.659$ $r_2=3554.227$ $r_3=4281.804$
22.	$a=10978.38$ $b=9662.8$ $A=68^{\circ}3'31.5''$	$R=5917.834$ $B=54^{\circ}43'38.54''$ $C=57^{\circ}12'49.96''$ $c=9950.22$	$r=2915.29$ $r_1=10328.493$ $r_2=7916.244$ $r_3=8341.887$
23.	$a=719.29$ $b=698.76$ $A=57^{\circ}41'11.7''$	$R=425.5465$ $B=55^{\circ}11'11.44''$ $C=67^{\circ}7'36.86''$ $c=784.17$	$r=210.2832$ $r_1=606.4242$ $r_2=575.4812$ $r_3=730.5638$
24.	$a=8625.87$ $b=7818.03$ $A=67^{\circ}37'52.48''$	$R=4663.873$ $B=56^{\circ}56'42.14''$ $C=55^{\circ}25'25.38''$ $c=7680.2$	$r=2301.68$ $r_1=8079.603$ $r_2=6541.669$ $r_3=6335.902$

$$\Delta_2 = 234.7964$$

$$\Delta_1 = 1000.0007$$

$$\Delta_2 = 603.2828$$

$$\Delta_3 = 1598.853$$

EXAMPLES VIII.—Continued.

DATA.	ANSWERS.
25. $a=11878.92$ $b=10630.44$ $A=63^{\circ}27'15.2''$	$R=6639.403$ $B=53^{\circ}10'59.98''$ $C=63^{\circ}21'44.82''$ $c=11869.4$ $r=3283.28$ $r_1=10627.62$ $r_2=8604.074$ $r_3=10608.6$
26. $a=8129.16$ $b=9246.77$ $B=63^{\circ}57'31.52''$	$R=5145.795$ $A=52^{\circ}10'28.76''$ $C=63^{\circ}51'59.72''$ $c=9239.49$ $r=2535.525$ $r_1=6515.731$ $r_2=8308.923$ $r_3=8294.053$
27. $a=8704.287$ $b=7895.319$ $A=67^{\circ}34'13.06''$	$R=4708.338$ $B=56^{\circ}58'32.84''$ $C=55^{\circ}27'14.1''$ $c=7756.236$ $r=2324.09$ $r_1=8147.836$ $r_2=6608.74$ $r_3=6400.864$
28. $a=2777.331$ $b=2028.932$ $A=81^{\circ}2'39.88''$	$R=1405.803$ $B=46^{\circ}11'20.42''$ $C=52^{\circ}45'59.7''$ $c=2238.537$ $r=636.8488$ $r_1=3010.775$ $r_2=1502.031$ $r_3=1747.254$
29. $a=6004.551$ $b=4940.314$ $A=52^{\circ}16'48.72''$	$R=3795.486$ $B=40^{\circ}36'10.9''$ $C=87^{\circ}7'0.38''$ $c=7581.363$ $r=1599.1825$ $r_1=4545.994$ $r_2=3426.808$ $r_3=8808.323$
30. $a=72027.91$ $b=63926$ $A=76^{\circ}22'16.08''$	$R=37057.4$ $B=59^{\circ}36'5.30''$ $C=44^{\circ}1'38.62''$ $c=51509.95$ $r=17070.53$ $r_1=73721.52$ $r_2=53682.39$ $r_3=37896.22$
31. $a=10301.99$ $b=9205.4$ $A=61^{\circ}16'11.5''$	$R=5874.135$ $B=51^{\circ}35'13.32''$ $C=67^{\circ}8'35.18''$ $c=10825.782$ $r=2880.918$ $r_1=8981.956$ $r_2=7329.696$ $r_3=10065.804$
32. $a=72027.91$ $b=58443.05$ $A=86^{\circ}18'43.14''$	$R=36088.7$ $B=54^{\circ}4'4.86''$ $C=39^{\circ}37'12''$ $c=46027$ $r=15209.16$ $r_1=82743.85$ $r_2=45031.10$ $r_3=31788.98$
33. $a=12785.2$ $b=11688.61$ $A=69^{\circ}4'52.18''$	$R=6843.687$ $B=58^{\circ}38'46.20''$ $C=52^{\circ}16'21.62''$ $c=10825.782$ $r=3348.419$ $r_1=12148.782$ $r_2=9913.952$ $r_3=8660.432$

IX.—Triangles in which two sides and the angle opposite the lesser are given. Ambiguous Case.

Let the two given lengths be a, b ($a > b$), and B the given angle. First calculate R by the equation $2R = b/\sin B$; then $\sin A$ may be found from $\sin A = a/2R$, but either the acute or the obtuse angle having this sine may be taken, and there are two triangles having the assigned data in all cases except when $A = 90^{\circ}$. Thus there will be two values of A , supplementary to each other. We shall denote the acute angle by A_1 , the obtuse by A_2 . The equation $A + B + C = 180^{\circ}$ supplies two corresponding values of C , which we denote by C_1, C_2 . The work is otherwise exactly the same as in VIII. The equation $b^2 = a^2 + c^2 - 2ac \cos B$ shows us that, if c_1, c_2 be the two values of c , $2a + c_1 = c_2 \cos B$, $c_1 c_2 = a^2 - b^2 \equiv (a - b)(a + b)$, and either of these may be used

to check the work. Since the angle B is common to both triangles, this angle should be used in calculating r , r_1 , r_2 , r_3 for the two triangles.

EXAMPLE WORKED OUT.

DATA.	$\log b = 4.1673297$	$\log a = 4.2273377$	2760(20.44"
$a = 16878.65$	$-\log \sin B = 1.9375306$	$-\log 2R = 4.2297991$	$A_1 = 83^\circ 54' 20.44''$
$b = 14700.42$	$\log 2R = 4.2297991$	$L \sin A = 9.9975386$	$C_1 = 36^\circ 5' 39.56''$
$B = 60^\circ$	842	$83^\circ 54' +$	$A_2 = 96^\circ 5' 39.56''$
	16974.58	135	$C_2 = 23^\circ 54' 20.44''$
	149	40	
	$\log 2R = 4.2297991$	1733	$\log 2R = 4.2297991$
	$\log \sin C_1 = 1.7700808$	36-1039 8	$-\log \sin C_2 = 1.6070068$
	1143	3.6-104	971
	$\log r_1 = 4.0000002$	-.04- 1 2	$\log c_2 = 3.8375030$
			01
	10000.005		6878.645
			29
	$\log (a-b) = 3.81038$	$\log \tan \frac{A}{2} = 1.7614394$	
	$\log (a+b) = 4.4993994$	$\log \cot \frac{1}{2} B = 2.385606$	
	$\log (a^2 - b^2) = 7.8375032$		
	$\log r_1 c_2 = 7.8375032$		
$a = 16878.65$	$s_1 - a = 3910.89$	$\log = 3.5922756$	
$b = 14700.42$	$s_1 - b = 6089.12$	$\log = 3.7845545$	
$c = 10000.01$	$s_1 - c = 10789.53$	$\log = 4.0330024$	
	$s_1 = 20789.54$	$\log = 4.3178449$	
$2s_1 = 41579.08$			
$s_1 = 20789.54$			
I.	$\log r$	$\log r_1$	$\log r_2$
	3.7845545	4.0330024	4.3178449
	1.7614394	.2385606	1.7614394
	3.5459939	4.2715630	4.0792843
	871	28	536
	3515.555	18088.01	12002.85
	68	2	307
			6773.86
			38
			4R = 33949.165
	$a = 16878.65$	$s_2 - a = 2350.21$	$\log = 3.3711067$
	$b = 14700.42$	$s_2 - b = 4528.44$	$\log = 3.6559486$
	$c_2 = 6878.65$	$s_2 - c_2 = 12350.21$	$\log = 4.0916743$
		$s_2 = 19228.86$	$\log = 4.2839535$
$2s_2 = 38457.72$			
$s_2 = 19228.86$			
II.	$\log r$	$\log r_1$	$\log r_2$
	3.6559486	4.0916743	4.2839535
	1.7614394	.2385606	1.7614394
	3.4173880	4.3302349	4.0453929
	720	11	621
	2614.497	21391.19	11101.79
	160	38	308
			4070.683
			89
			4R = 33949.166
ANSWERS (I.).			
$R = 8487.29$	$r = 3515.555$	$R = 8487.29$	$r = 2614.497$
$A = 83^\circ 54' 20.44''$	$r_1 = 18688.01$	$A = 96^\circ 5' 39.56''$	$r_1 = 21391.19$
$C = 36^\circ 5' 39.56''$	$r_2 = 12002.85$	$C = 23^\circ 54' 20.44''$	$r_2 = 11101.79$
$c = 10000.005$	$r_3 = 6773.86$	$c = 6878.645$	$r_3 = 4070.683$
ANSWERS (II.).			

EXAMPLES IX.

DATA.

ANSWERS (1).

ANSWERS (2).

1. $a=3830.683$ $b=3396.549$ $B=62^{\circ}27'27.4''$	$R=1015.342$ $A=90^{\circ}$ $C=27^{\circ}32'32.6''$ $c=1771.326$	$r=668.596$ $r_1=4499.279$ $r_2=2727.953$ $r_3=1102.73$	Same as (1)	Same as (1)
2. $a=270.5115$ $b=116.5808$ $B=25^{\circ}31'43.6''$	$R=135.2558$ $A=90^{\circ}$ $C=64^{\circ}28'16.4''$ $c=244.1011$	$r=45.0852$ $r_1=315.5967$ $r_2=71.4956$ $r_3=199.0159$	Same as (1)	Same as (1)
3. $a=3650.401$ $b=2107.56$ $B=30^{\circ}$	$R=2107.56$ $A=60^{\circ}$ $C=90^{\circ}$ $c=4215.12$	$r=771.4205$ $r_1=2878.9805$ $r_2=1336.1395$ $r_3=4986.5405$	$R=2107.56$ $A=120^{\circ}$ $C=30^{\circ}$ $c=2107.56$	$r=489.0611$ $r_1=1051.7405$ $r_2=1033.78$ $r_3=1053.78$
4. $a=24150.16$ $b=22202.21$ $B=50^{\circ}59'37.5''$	(1) $\Delta_1=2534.15$ (2) $\Delta_1=18006.17$	$r=6918.088$ $r_1=20222.02$ $r_2=17506.51$ $r_3=26332.42$	$R=14285.71$ $A=122^{\circ}18'2.5''$ $C=6^{\circ}42'20''$ $c=3336.198$	$r=1260.025$ $r_1=45099.07$ $r_2=11848.4$ $r_3=1455.472$
5. $a=10316.36$ $b=6685.65$ $B=39^{\circ}53'33''$	(1) $\Delta_1=1873.9565$ (2) $\Delta_1=2672.551$	$r=2269.206$ $r_1=11197.104$ $r_2=4695.632$ $r_3=7225.329$	$R=5212.165$ $A=8^{\circ}44'47.09''$ $C=58^{\circ}21'39.91''$ $c=887.497$	$r=1020.904$ $r_1=13841.724$ $r_2=4347.183$ $r_3=4580.709$
6. $a=10316.36$ $b=6683.635$ $B=39^{\circ}53'33''$	(1) $\Delta_1=1877.5675$ (2) $\Delta_1=2664.942$	$r=2267.026$ $r_1=11213.669$ $r_2=4692.57$ $r_3=7203.212$	$R=5210.6665$ $A=8^{\circ}51'55.83''$ $C=58^{\circ}14'31.17''$ $c=8860.933$	$r=1023.817$ $r_1=13819.64$ $r_2=4349.64$ $r_3=4597.275$
7. $a=346.4537$ $b=301.3984$ $B=60^{\circ}153.4''$	$R=173.95725$ $A=8^{\circ}44'52.3''$ $C=35^{\circ}13'14.3''$ $c=200.6515$	$r=70.97449$ $r_1=387.0413$ $r_2=245.09745$ $r_3=134.6647$	$R=173.95725$ $A=95^{\circ}15'7.7''$ $C=24^{\circ}42'58.9''$ $c=145.3837$	$r=55.00991$ $r_1=434.8742$ $r_2=229.1328$ $r_3=86.8318$
8. $a=3156.045$ $b=3013.734$ $B=72^{\circ}13'57.6''$	$R=1582.339$ $A=8^{\circ}46'1.3''$ $C=22^{\circ}01.1''$ $c=1185.525$	$r=484.4273$ $r_1=3415.513$ $r_2=2683.396$ $r_3=714.8735$	$R=1582.339$ $A=94^{\circ}13'58.7''$ $C=13^{\circ}32'3.7''$ $c=740.625$	$r=322.116$ $r_1=3720.385$ $r_2=252.085$ $r_3=410.002$

EXAMPLES IX.—Continued.

DATA.	ANSWERS (1).			ANSWERS (2).		
	Δ	Δ_1	Δ_2	Δ	Δ_1	Δ_2
9. $a=7631.229$ $b=4973.381$ $B=39^\circ 53' 46.45''$	$\Delta=38^\circ 6' 9.8''$ $\Delta_1=79^\circ 47' 32.86''$ $\Delta_2=60^\circ 18' 40.69''$ $c=6736.086$	$r=1704.741$ $r_1=8684.589$ $r_2=3509.805$ $r_3=5618.257$	$r=1704.741$ $r_1=8684.589$ $r_2=3509.805$ $r_3=5618.257$	$\Delta=38^\circ 6' 9.8''$ $\Delta_1=79^\circ 47' 32.86''$ $\Delta_2=60^\circ 18' 40.69''$ $c=6736.086$	$r=1704.741$ $r_1=8684.589$ $r_2=3509.805$ $r_3=5618.257$	$r=1704.741$ $r_1=8684.589$ $r_2=3509.805$ $r_3=5618.257$
10. $a=4731.231$ $b=2718.108$ $B=32^\circ 57' 6''$	$\Delta=2560.001$ $\Delta_1=67^\circ 31' 40.31''$ $\Delta_2=60^\circ 24' 22.09''$ $c=5048.411$	$r=1014.619$ $r_1=4177.595$ $r_2=1795.749$ $r_3=5281.324$	$r=1014.619$ $r_1=4177.595$ $r_2=1795.749$ $r_3=5281.324$	$\Delta=2560.001$ $\Delta_1=67^\circ 31' 40.31''$ $\Delta_2=60^\circ 24' 22.09''$ $c=5048.411$	$r=1014.619$ $r_1=4177.595$ $r_2=1795.749$ $r_3=5281.324$	$r=1014.619$ $r_1=4177.595$ $r_2=1795.749$ $r_3=5281.324$
11. $a=481.2793$ $b=360.8528$ $B=46^\circ 44' 10.4''$	$\Delta=247.7683$ $\Delta_1=76^\circ 13' 21.66''$ $\Delta_2=57^\circ 27' 9.4''$ $c=415.7854$	$r=115.8427$ $r_1=493.3678$ $r_2=71.7591$ $r_3=341.7890$	$r=115.8427$ $r_1=493.3678$ $r_2=71.7591$ $r_3=341.7890$	$\Delta=247.7683$ $\Delta_1=76^\circ 13' 21.66''$ $\Delta_2=57^\circ 27' 9.4''$ $c=415.7854$	$r=115.8427$ $r_1=493.3678$ $r_2=71.7591$ $r_3=341.7890$	$r=115.8427$ $r_1=493.3678$ $r_2=71.7591$ $r_3=341.7890$
12. $a=59406.24$ $b=22733.78$ $B=22^\circ 30'$	$\Delta=29703.12$ $\Delta_1=90^\circ$ $\Delta_2=67^\circ 30'$ $c=54884.2$	$r=9105.87$ $r_1=68512.11$ $r_2=13627.91$ $r_3=45778.33$	$r=9105.87$ $r_1=68512.11$ $r_2=13627.91$ $r_3=45778.33$	$\Delta=29703.12$ $\Delta_1=90^\circ$ $\Delta_2=67^\circ 30'$ $c=54884.2$	$r=9105.87$ $r_1=68512.11$ $r_2=13627.91$ $r_3=45778.33$	$r=9105.87$ $r_1=68512.11$ $r_2=13627.91$ $r_3=45778.33$
13. $a=964701$ $b=955078$ $B=59^\circ 59' 23.7''$	(1) $\Delta_1=9628.197$ (2) $\Delta_1=542988.5$	$r=551470.5$ $r_1=61^\circ 0' 18.1''$ $r_2=59^\circ 0' 18.13''$ $r_3=91543.9$	$r=551470.5$ $r_1=61^\circ 0' 18.1''$ $r_2=59^\circ 0' 18.13''$ $r_3=91543.9$	$\Delta=551470.5$ $\Delta_1=61^\circ 0' 18.1''$ $\Delta_2=59^\circ 0' 18.13''$ $c=19539.84$	$r=551470.5$ $r_1=61^\circ 0' 18.1''$ $r_2=59^\circ 0' 18.13''$ $r_3=91539.84$	$r=551470.5$ $r_1=61^\circ 0' 18.1''$ $r_2=59^\circ 0' 18.13''$ $r_3=91539.84$
14. $a=337.573$ $b=294.0084$ $B=60^\circ$	(1) $\Delta_1=70.311$ $\Delta_2=70.311$ (2) $\Delta_1=105.17894$	$r=169.74584$ $r_1=83^\circ 54' 20.44''$ $r_2=36^\circ 5' 39.56''$ $r_3=200$	$r=169.74584$ $r_1=83^\circ 54' 20.44''$ $r_2=36^\circ 5' 39.56''$ $r_3=200$	$\Delta=169.74584$ $\Delta_1=83^\circ 54' 20.44''$ $\Delta_2=36^\circ 5' 39.56''$ $c=137.57292$	$r=169.74584$ $r_1=83^\circ 54' 20.44''$ $r_2=36^\circ 5' 39.56''$ $r_3=137.57292$	$r=169.74584$ $r_1=83^\circ 54' 20.44''$ $r_2=36^\circ 5' 39.56''$ $r_3=137.57292$
15. $a=6142.887$ $b=6127.56$ $B=59^\circ 45' 12''$	(1) $\Delta_1=30.539$ (2) $\Delta_1=6138.099$	$r=3546.597$ $r_1=60^\circ$ $r_2=60^\circ 14' 48''$ $r_3=6158.099$	$r=3546.597$ $r_1=60^\circ$ $r_2=60^\circ 14' 48''$ $r_3=6158.099$	$\Delta=3546.597$ $\Delta_1=60^\circ$ $\Delta_2=60^\circ 14' 48''$ $c=30.539$	$r=3546.597$ $r_1=60^\circ$ $r_2=60^\circ 14' 48''$ $r_3=30.539$	$r=3546.597$ $r_1=60^\circ$ $r_2=60^\circ 14' 48''$ $r_3=30.539$
16. $a=775.2704$ $b=674.2277$ $B=60^\circ$	(1) $\Delta_1=166.874$ $\Delta_2=325.9911$ (2) $\Delta_1=235.7022$ $\Delta_2=149.2793$	$r=389.2655$ $r_1=85^\circ 45' 15''$ $r_2=33^\circ 14' 45''$ $r_3=449.2793$	$r=389.2655$ $r_1=85^\circ 45' 15''$ $r_2=33^\circ 14' 45''$ $r_3=449.2793$	$\Delta=389.2655$ $\Delta_1=85^\circ 45' 15''$ $\Delta_2=33^\circ 14' 45''$ $c=325.9911$	$r=389.2655$ $r_1=85^\circ 45' 15''$ $r_2=33^\circ 14' 45''$ $r_3=325.9911$	$r=389.2655$ $r_1=85^\circ 45' 15''$ $r_2=33^\circ 14' 45''$ $r_3=325.9911$

EXAMPLES IX.—Continued.

DATA.	ANSWERS (1).		ANSWERS (2).	
17. $a=701.37$ $b=466.3$ $B=54^{\circ} 9.25''$	$A=411.7826$ $A=58^{\circ} 23.20.5''$ $C=67^{\circ} 36.30.25''$ $c=761.47$	$r=202.9399$ $r_1=594.8341$ $r_2=542.4556$ $r_3=712.7807$	$A=411.7826$ $A=121^{\circ} 36.39.5''$ $C=4^{\circ} 23.11.25''$ $c=629.89$	$r=24.9832$ $r_1=1286.219$ $r_2=364.4987$ $r_3=27.3955$
18. $a=393.75$ $b=391.54$ $B=64^{\circ} 46.15.35''$	$A=216.4135$ $A=65^{\circ} 27.58.58''$ $C=49^{\circ} 35.46.06''$ $c=330.41$	$r=125.4844$ $r_1=358.589$ $r_2=353.8239$ $r_3=258.7252$	$A=216.4135$ $A=114^{\circ} 32.1.42''$ $C=0^{\circ} 41.43.22''$ $c=3.25254$	$r=2.36661$ $r_1=614.9157$ $r_2=250.7062$ $r_3=2.39848$
19. $a=5265.68$ $b=5822.3$ $B=65^{\circ} 28.2.86''$	$A=2650.415$ $A=83^{\circ} 23.53.3''$ $C=31^{\circ} 8.3.84''$ $c=2740.78$	$r=1023.121$ $r_1=5714.817$ $r_2=4123.293$ $r_3=1786.97$	$A=2650.415$ $A=96^{\circ} 36.6.7''$ $C=17^{\circ} 55.50.44''$ $c=1631.947$	$r=667.0308$ $r_1=5577.292$ $r_2=376.902$ $r_3=924.4924$
20. $a=688.9$ $b=627.77$ $B=64^{\circ} 16.8.94''$	$A=348.4346$ $A=81^{\circ} 19.36.02''$ $C=34^{\circ} 24.15.04''$ $c=393.75$	$r=142.8642$ $r_1=734.6451$ $r_2=537.1917$ $r_3=264.7658$	$A=348.4346$ $A=98^{\circ} 40.23.98''$ $C=17^{\circ} 32.7.05''$ $c=204.414$	$r=83.40002$ $r_1=885.3568$ $r_2=477.7268$ $r_3=114.0542$
21. $a=313.885$ $b=165.205$ $B=35^{\circ} 44.5.06''$	$A=190.2383$ $A=55^{\circ} 35.10.03''$ $C=98^{\circ} 40.44.32''$ $c=376.12$	$r=59.94535$ $r_1=225.3852$ $r_2=97.67535$ $r_3=497.8303$	$A=190.2383$ $A=124^{\circ} 24.49.38''$ $C=29^{\circ} 51.5.56''$ $c=189.384$	$r=38.61218$ $r_1=634.1209$ $r_2=76.35015$ $r_3=89.09454$
22. $a=10978.38$ $b=9662.8$ $B=54^{\circ} 43.38.54''$	$A=5917.834$ $A=68^{\circ} 31.31.5''$ $C=57^{\circ} 12.49.96''$ $c=9950.22$	$r=2915.29$ $r_1=10328.493$ $r_2=7916.244$ $r_3=8341.887$	$A=5917.834$ $A=111^{\circ} 56.28.5''$ $C=13^{\circ} 15.52.96''$ $c=2729.1$	$r=1046.656$ $r_1=17304.79$ $r_2=6047.607$ $r_3=1365.556$
23. $a=1908.237$ $b=633.911$ $B=39^{\circ} 20.21.35''$	$A=500$ $A=36^{\circ} 35.50.4''$ $C=54^{\circ} 3.48.25''$ $c=809.0609$	$r=209.8239$ $r_1=1150.473$ $r_2=436.4176$ $r_3=622.9357$	$A=500$ $A=93^{\circ} 24.9.6''$ $C=47^{\circ} 15.29.05''$ $c=734.4185$	$r=106.375$ $r_1=1255.73$ $r_2=422.9687$ $r_3=517.6794$
24. $a=4457.15$ $b=4164.894$ $B=54^{\circ} 18.24''$	$A=2573.337$ $A=60''$ $C=65^{\circ} 58.41.76''$ $c=4700.928$	$r=1272.674$ $r_1=3846.01$ $r_2=3395.787$ $r_3=4324.224$	$A=2573.337$ $A=120''$ $C=5^{\circ} 58.41.76''$ $c=536.0318$	$r=211.116$ $r_1=7931.125$ $r_2=2334.23$ $r_3=239.1052$

X.—Tetrahedra, the lengths of the edges being given.

In a tetrahedron $OABC$ let the lengths of OA , OB , OC be denoted by a , b , c ; the lengths of the respectively opposite edges BC , CA , AB by x , y , z ; the dihedral angles opposite to a , b , c , x , y , z respectively by A , B , C , X , Y , Z . Also, let any angle of a triangular face which is opposite to OA or BC be denoted by α , one opposite to OB or CA by β , one opposite to OC or AB by γ ; all the angles at O having the suffix 1, those at A the suffix 2, those at B the suffix 3, and those at C the suffix 4. Then, in each of the four triangular faces we have the lengths of the three sides given, and may calculate the angles as in VI. In the face ABC opposite O the lengths are x , y , z , the angles α_3 , β_3 , γ_1 ; in the face opposite A the lengths are x , b , c , the angles α_1 , β_4 , γ_3 ; in the face opposite B the lengths are a , y , c , the angles α_4 , β_1 , γ_2 ; lastly, in the face opposite C the lengths are a , b , z , the angles α_2 , β_2 , γ_4 . The sums of the lengths are denoted by $2s_1$, $2s_2$, $2s_3$, $2s_4$. The sum of the plane angles at O is similarly $2\sigma_1$, and the sums of the angles at A , B , C are $2\sigma_2$, $2\sigma_3$, $2\sigma_4$. The volume V may be found from each of the equations

$$\begin{aligned} 3V &= abc \sqrt{\sin \sigma_1 \sin (\sigma_1 - \alpha_1)} \sqrt{\sin (\sigma_1 - \beta_1) \sin (\sigma_1 - \gamma_1)} \\ &= ayz \sqrt{\sin \sigma_2 \sin (\sigma_2 - \alpha_2)} \sqrt{\sin (\sigma_2 - \beta_2) \sin (\sigma_2 - \gamma_2)} \\ &= abx \sqrt{\sin \sigma_3 \sin (\sigma_3 - \alpha_3)} \sqrt{\sin (\sigma_3 - \beta_3) \sin (\sigma_3 - \gamma_3)} \\ &= xyz \sqrt{\sin \sigma_4 \sin (\sigma_4 - \alpha_4)} \sqrt{\sin (\sigma_4 - \beta_4) \sin (\sigma_4 - \gamma_4)}. \end{aligned}$$

The most convenient manner of arranging the work will be obvious on inspecting the one calculated. The dihedral angles at O are X , Y , Z , and these are to be determined by the equations

$$\begin{aligned} \tan \frac{1}{2} X \sin (\sigma_1 - \alpha_1) &= \tan \frac{1}{2} Y \sin (\sigma_1 - \beta_1) = \tan \frac{1}{2} Z \sin (\sigma_1 - \gamma_1) \\ &= \sqrt{\frac{\sin (\sigma_1 - \alpha_1) \sin (\sigma_1 - \beta_1) \sin (\sigma_1 - \gamma_1)}{\sin \sigma_1}}, \end{aligned}$$

no fresh logarithms needing to be taken out. This part of the work is almost exactly the same as that of calculating the tangents of the half angles of a triangle when the sides are given. The dihedral angles at A are X , B , C ; at B are A , Y , C ; and at C are A , B , Z ; and are to be found in the same way from the angles with suffixes 2, 3, 4 respectively. Thus the volume is calculated from four different expressions, and each dihedral angle from two; and the agreement of the results is sufficient to guarantee the correctness of the calculations. From each tetrahedron we get the six parts of each of four plane triangles, and the six parts of each of four spherical triangles; and these may be used to supply any deficiency in any of the preceding sets of examples; or for practice in solving spherical triangles. To make these plane and spherical triangles complete, I have given always the values of all the plane angles of each face of every tetrahedron, or rather the halves of these angles. The order of magnitude of $A+X$, $B+Y$, $C+Z$ being always the same as that of $a+x$, $b+y$, $c+z$; that of $A-X$, $B-Y$, $C-Z$, the same as that of $a-x$, $b-y$, $c-z$; that of $A-X$, $Y-B$, $Z-C$, the same as that of $a-x$, $y-b$, $z-c$, etc.; the results should be tested accordingly. When

two terms of one system of lengths are equal, the two corresponding terms of the corresponding system of angles will be also equal. In many cases, the data will be found to satisfy one, or two, of such equations, and it will be seen that the calculated angles satisfy the corresponding equations, *quam prox.* Many other properties of tetrahedra are illustrated, attention being directed to such cases by a very brief entry in column of remarks. The tetrahedron in which a, b, c are the lengths of three conterminous edges, and x, y, z the lengths of the edges respectively opposite

to a, b, c , is here denoted by $\begin{Bmatrix} a & x \\ b & y \\ c & z \end{Bmatrix}$. The same tetrahedron will thus be also

denoted by $\begin{Bmatrix} a & x \\ y & b \\ z & c \end{Bmatrix}$, $\begin{Bmatrix} x & a \\ b & y \\ c & z \end{Bmatrix}$, or $\begin{Bmatrix} x & a \\ y & b \\ c & z \end{Bmatrix}$. When six lengths are given, there will

be thirty different tetrahedra having these for edges, but some of these may be impossible, even when any set of three of the lengths are sides of a real triangle. In dealing with such a system of tetrahedra, it seems most convenient to arrange them in five groups of six, the longest edge having the same edge opposite to it in all the tetrahedra of any one group.

The sums of the dihedral angles at O, A, B, C respectively are denoted by $2S_1, 2S_2, 2S_3, 2S_4$; and the order of magnitude of S_1, S_2, S_3, S_4 corresponds to that of s_1, s_2, s_3, s_4 . When $a - x = b - y = c - z$, $s_2 = s_3 = s_4$, $S_2 = S_3 = S_4$; and, if $s_2 > s_3 > s_4$, it will be observed that

$$\sin \sigma_2 \sin (\sigma_2 - \alpha_2) \sin (\sigma_2 - \beta_2) \sin (\sigma_2 - \gamma_2) < \sin \sigma_3 \sin (\sigma_3 - \alpha_3) \sin (\sigma_3 - \beta_3) \sin (\sigma_3 - \gamma_3) \\ < \sin \sigma_4 \sin (\sigma_4 - \alpha_4) \sin (\sigma_4 - \beta_4) \sin (\sigma_4 - \gamma_4);$$

also

$$\sin (\sigma_2 - \alpha_2) \sin (\sigma_2 - \beta_2) \sin (\sigma_2 - \gamma_2) / \sin \sigma_2 < \sin (\sigma_3 - \alpha_3) \sin (\sigma_3 - \beta_3) \sin (\sigma_3 - \gamma_3) / \sin \sigma_3 \\ < \sin (\sigma_4 - \alpha_4) \sin (\sigma_4 - \beta_4) \sin (\sigma_4 - \gamma_4) / \sin \sigma_4.$$

Many cases of maximum volume under certain conditions will be found.

From the equations of which $\frac{\partial V}{\partial a} = \frac{1}{8}ax \cot A$ is the type, we see that when a, x vary subject to the condition that $a+x$ is constant, the volume (V) will be a maximum when $A = X$; when subject to the condition that $a-x$ is constant, V will be a maximum when $A+X=180^\circ$; and similarly with the sum or difference of any pair of opposite edges. When $a, x, b+y, c+z$ are given, V will be a maximum when $B=Y, C=Z$, which involves $b=y, c=z$. When $a, x, b+y, c-z$ are given, V will be a maximum when $B=Y, C+Z=180^\circ$. As the lengths of the edges are varied so as to approach this maximum value, it appears that the variations of b and y do not alter $C+Z$; nor those of c and z affect $B-Y$. When $a, x, b-y, c-z$ are given, it should seem that V will be a maximum when $B+Y=C+Z=180^\circ$, but this maximum will be when b, y, c, z are infinite, and consequently V .

In any tetrahedron in which $\alpha_1 + \beta_1 + \gamma_1 = \alpha_2 + \beta_2 + \gamma_2 = \pi$, it will be found that

$$\gamma_1 + \alpha_1 - \beta_1 = \gamma_4 + \alpha_4 - \beta_4; \quad \alpha_1 + \beta_1 - \gamma_1 = \alpha_3 + \beta_3 - \gamma_3; \\ \gamma_2 + \alpha_2 - \beta_2 = \gamma_3 + \alpha_3 - \beta_3; \quad \alpha_2 + \beta_2 - \gamma_2 = \alpha_4 + \beta_4 - \gamma_4;$$

$$\begin{cases} a=3.5683, & x=4.9792 \\ b=3.6767, & y=5.0876 \\ c=4.2235, & z=5.6344 \end{cases}$$

Calculation of the Angles of the Faces.

$x = 4.9792$	$s_1 - a = 2.8714$	$\log(s_1 - a) = .4580937$	10.1750904	$90360(29.95)$	10.1750904	$30540(11.21)$	$\frac{1}{2}a = 27^\circ 31' 41.18''$
$y = 5.0876$	$s_1 - b = 2.7630$	$\log(s_1 - b) = .4413809$	$.4580937$	30020	$.4413809$	3300	$\frac{1}{2}b = 28^\circ 26' 30.04''$
$z = 5.6344$	$s_1 - c = 2.2162$	$\log(s_1 - c) = .3456090$	2484	2867	9.8201814	576	$\frac{1}{2}c = 34^\circ 1' 48.78''$
15.7012	$s_1 = 7.8506$	$-\log s_1 = -.8949028$	9.7169967	152	$34^\circ 2' - 53^\circ 3'$	31	$90^\circ 0' 0''$
$s_1 = 7.8506$		$2) \cdot 3501808$	$27^\circ 32' - 1^\circ 09' 33''$	3082	3017	509	
			3682	1506			
$x = 4.9792$	$s_2 - a = 1.4605$	$\log(s_2 - a) = .1645016$	10.0713130	14224.56	10.0713130	$17760(5.83)$	$\frac{1}{2}a = 38^\circ 53' 58.26''$
$y = 3.6767$	$s_2 - b = 2.7630$	$\log(s_2 - b) = .4413809$	$.1645016$	22	$.4413809$	2525	$\frac{1}{2}b = 23^\circ 55' 9.9''$
$z = 4.2235$	$s_2 - c = 2.2162$	$\log(s_2 - c) = .3456090$	45		9.7257040	87	$\frac{1}{2}c = 28^\circ 0' 5.81''$
12.8794	$s_2 = 6.4397$	$-\log s_2 = -.8688656$	9.9068114		$28^\circ 0' + 67^\circ 44'$		$90^\circ 0' 0''$
$s_2 = 6.4397$		$2) \cdot 1426259$	$35^\circ 54' - 25^\circ 55'$	3500	3047	296	
			74	237			
$x = 3.5683$	$s_3 - a = 2.8714$	$\log(s_3 - a) = .4580937$	10.0629230	$57780(22.59)$	10.0629230	$52440(17.02)$	$\frac{1}{2}a = 21^\circ 55' 39.6''$
$y = 5.0876$	$s_3 - b = 1.3521$	$\log(s_3 - b) = .1310088$	4580937	6620	$.1310088$	2163	$\frac{1}{2}b = 40^\circ 31' 37.40''$
$z = 4.2235$	$s_3 - c = 2.2162$	$\log(s_3 - c) = .3456090$	126	1504	9.9319142	63	$\frac{1}{2}c = 27^\circ 32' 42.96''$
12.8794	$s_3 = 6.4397$	$-\log s_3 = -.8688656$	9.6048293	225	$40^\circ 32' - 20^\circ 105'$	97173140	$50^\circ 0' 0''$
$s_3 = 6.4397$		$21^\circ 56' - 36^\circ 47'$	9329	2558	3081	874	
			1236	963			
$x = 3.5683$	$s_4 - a = 2.8714$	$\log(s_4 - a) = .4580937$	9.9982834	$106200(26.82)$	9.9982834	$48120(18.62)$	$\frac{1}{2}a = 19^\circ 7' 51.82''$
$y = 3.6767$	$s_4 - b = 2.7630$	$\log(s_4 - b) = .4413809$	$.4580937$	2702	$.4413809$	22280	$\frac{1}{2}b = 19^\circ 49' 56.81''$
$z = 5.6344$	$s_4 - c = 2.8053$	$\log(s_4 - c) = .19050577$	9.5401807	3566	1.9059577	1608	$\frac{1}{2}c = 51^\circ 241.37''$
12.8794	$s_4 = 6.4397$	$-\log s_4 = -.8688656$	9.5401807	99	$10^\circ 09' 23' 57''$	58	$90^\circ 0' 0''$
$s_4 = 6.4397$		$19^\circ 8' - 40^\circ 82'$	2453	3959	$5^\circ 31' + 40^\circ 59'$	2584	
			556	1770	802		

Calculation of the Volume.

$\alpha_1 = 77^\circ 47' 56.52''$ $\beta_1 = 81^\circ 3' 14.80''$ $\gamma_1 = 102^\circ 5' 22.74''$	$\alpha_2 = 55^\circ 3' 22.36''$ $\beta_2 = 39^\circ 38' 53.62''$ $\gamma_2 = 55^\circ 5' 25.92''$	$341 : 6 - \frac{1}{2} + \frac{1}{16}$ 305 7 -17 +8	$\alpha_3 = 38^\circ 15' 43.64''$ $\beta_3 = 56^\circ 53' 0.08''$ $\gamma_3 = 56^\circ 0' 11.62''$	$325 : 53 + \frac{1}{2}$ 138 4 9 2 2 3	$\alpha_4 = 43^\circ 51' 19.28''$ $\beta_4 = 46^\circ 11' 51.86''$ $\gamma_4 = 68^\circ 3' 37.56''$	$244 : 41 \frac{1}{2}$ 97 4 1 6
$260^\circ 56' 34.06''$ $\sigma_1 = 130^\circ 28' 17.03''$	$149^\circ 47' 41.90''$ $\sigma_2 = 74^\circ 53' 50.95''$	$3501 : 30 = 1150$ -1.5 -87 5 + .09= 5 3	$151^\circ 8' 55.34''$ $\sigma_3 = 75^\circ 34' 27.67''$	$1658 : 40 = 1105$ 4= 110 8 .03=	$158^\circ 6' 48.70''$ $\sigma_4 = 79^\circ 3' 24.35''$	$1791 : 5 = 149$ 3 .06= 1 8 01= 3
$\sigma_1 - \alpha_1 = 52^\circ 40' 20.51''$ $\sigma_1 - \beta_1 = 49^\circ 25' 2.23''$ $\sigma_1 - \gamma_1 = 28^\circ 22' 54.29''$	$\sigma_2 - \alpha_2 = 19^\circ 50' 28.59''$ $\sigma_2 - \beta_2 = 35^\circ 14' 57.33''$ $\sigma_2 - \gamma_2 = 19^\circ 48' 25.03''$	$1788 : 54 = 1609$ 3= 89 4 -3= 8 9 .03= 9	$37^\circ 18' 44.03''$ $\sigma_3 - \alpha_3 = 18^\circ 41' 27.59''$ $\sigma_3 - \beta_3 = 19^\circ 34' 16.05''$ $\sigma_3 - \gamma_3 = 19^\circ 34' 16.05''$	$3738 : 20 = 1246$ 7.5= 467 3 .09= 5 6 5 6	$35^\circ 12' 5.07''$ $\sigma_4 - \alpha_4 = 32^\circ 51' 32.40''$ $\sigma_4 - \beta_4 = 10^\circ 59' 46.79''$ $\sigma_4 - \gamma_4 = 10^\circ 59' 46.79''$	$1956 : 30 = 978$ 2.4= 78 2 .09= 2 9
$\sigma_1 = 130^\circ 28' 17.03''$ $86^\circ - \sigma_1 = 49^\circ 31' 42.97''$	$\sigma_2 = 74^\circ 53' 50.95''$	$3508 : 20 = 1169$ 5= 292 3 -03= 1 8	$\sigma = 75^\circ 34' 27.67''$	$3553 : 15 = 888$ 1= 59 2 .03= 3	$\sigma = 79^\circ 3' 24.35''$	$6504 : 48 = 5203$ -1.2= -130 1 -01= -1 1
$L \sin \sigma_1 = 9.8811534$ $L \sin (\sigma_1 - \alpha_1) = 9.9004331$ $L \sin (\sigma_1 - \beta_1) = 9.8805952$ $L \sin (\sigma_1 - \gamma_1) = 9.6767963$	$L \sin \sigma_2 = 9.9847059$ $L \sin (\sigma_2 - \alpha_2) = 9.5305650$ $L \sin (\sigma_2 - \beta_2) = 9.7011063$ $L \sin (\sigma_2 - \gamma_2) = 9.5298638$	289 5 1668 3 1708 4 1463 4	$L \sin \sigma = 9.9860720$ $L \sin (\sigma - \alpha) = 9.7824643$ $L \sin (\sigma - \beta) = 9.5056077$ $L \sin (\sigma - \gamma) = 9.5249196$	149 9 1216 6 1718 9 950 5	$L \sin \sigma = 9.9920201$ $L \sin (\sigma - \alpha) = 9.7607483$ $L \sin (\sigma - \beta) = 9.7343529$ $L \sin (\sigma - \gamma) = 9.2799484$	99 0 151 4 1059 1 5072 0
$2) 1.3392138$	$2) 2.8067540$	5129 6	$2) 3.7994672$	4035 9	$2) 2.7677078$	6381 5
1.6696669 $\log \sigma = .5524614$ $\log \beta = .5054582$ $\log \gamma = .6256725$ $-\log 3 = .4771213$ $\log V = .9360777$	1.4033770 $\log \sigma = .5524014$ $\log \beta = .7065130$ $\log \gamma = .7508477$ $-\log 3 = .4771213$ $\log V = .9360778$		1.3997336 $\log \sigma = .6971596$ $\log \beta = .5654582$ $\log \gamma = .7508477$ $-\log 3 = .4771213$ $\log V = .9360778$		1.3838539 $\log \sigma = .6971596$ $\log \beta = .7065130$ $\log \gamma = .6256725$ $-\log 3 = .4771213$ $\log V = .9360777$	$1=8.631331$ $1=8.631331$
						$1=8.631332$ $1=8.631332$

Calculation of the Dihedral Angles.

$-L \sin \sigma_1 = -9.8812366$	8	19.7883763	19.7883763	19.7883763	-9.9847348	5	19.4186421	19.4186421
$L \sin(\sigma_1 - \alpha_1) = 9.9004660$	2	9.9004660	9.8805092	9.6770079	9.5307318	3	9.5307318	9.5300101
$L \sin(\sigma_1 - \beta_1) = 9.8805092$	2				9.7612771	4		
$L \sin(\sigma_1 - \gamma_1) = 9.6770079$	4	$L \tan \frac{1}{2} X$	$L \tan \frac{1}{2} Y$	$L \tan \frac{1}{2} Z$	9.5300101	4	$L \tan \frac{1}{2} Y$	$L \tan \frac{1}{2} C$
2) 19.5767525		9.8879103	9.9078671	10.1113684	2) 18.8372842	6	9.8879103	9.8886320
$-L \sin \sigma_2 = -9.9860869$	9	19.4136466	19.4136466	19.4136466	-9.9920300		19.3918239	19.3918239
$L \sin(\sigma_2 - \alpha_2) = 9.7825859$	6	9.7825860	9.5037796	9.5250146	9.7607634	4	9.7607634	9.7344588
$L \sin(\sigma_2 - \beta_2) = 9.5037795$	9				9.7344588	1		9.2804556
$L \sin(\sigma_2 - \gamma_2) = 9.5250146$	5	$L \tan \frac{1}{2} X$	$L \tan \frac{1}{2} Y$	$L \tan \frac{1}{2} C$	9.2804556		$L \tan \frac{1}{2} Y$	$L \tan \frac{1}{2} Z$
2) 18.8272932	1	9.6310666	9.9078670	9.8886320	18.7836478	5	9.6310665	10.1113683
$L \tan \frac{1}{2} A = 9.6310666$		33.40(9.32	$L \tan \frac{1}{2} B = 9.6573651$	900(27	$L \tan \frac{1}{2} C = 9.8886320$		3960(1.52	
$23^{\circ} 9' + \left\{ \begin{array}{l} 032 \\ 554 \end{array} \right\}$		1803	$24^{\circ} 26' + \left\{ \begin{array}{l} 36 \\ 15 \end{array} \right\}$	229	37' 44' - $\left\{ \begin{array}{l} 86 \\ 2611 \end{array} \right\}$		1349	
$L \tan \frac{1}{2} X = 9.8879103$		56	3353	15			43	
$37^{\circ} 41' + \left\{ \begin{array}{l} 6830 \\ 1608 \end{array} \right\}$		32940(12.62	$L \tan \frac{1}{2} Y = 9.9078671$	8760(3.39	$L \tan \frac{1}{2} Z = 10.1113684$		4200(1.61	
		549	$38^{\circ} 58' + \left\{ \begin{array}{l} 525 \\ 2584 \end{array} \right\}$	1008	$52^{\circ} 16' + \left\{ \begin{array}{l} 14 \\ 2611 \end{array} \right\}$		1589	
		41		233			70	

ANSWERS.

$\frac{1}{2} A = 23^{\circ} 9' 9.50''$	$\frac{1}{2} B = 24^{\circ} 26' 0.25''$	$\frac{1}{2} C = 37^{\circ} 43' 58.48''$	$\sigma_1 = 130^{\circ} 28' 17.03''$
$\frac{1}{2} X = 37^{\circ} 41' 12.63''$	$\frac{1}{2} Y = 38^{\circ} 58' 3.38''$	$\frac{1}{2} Z = 52^{\circ} 16' 1.61''$	$\sigma_2 = 74^{\circ} 53' 50.95''$
$\frac{1}{2} (A + X) = 60^{\circ} 50' 22.13''$	$\frac{1}{2} (B + Y) = 63^{\circ} 24' 3.63''$	$\frac{1}{2} (Z + C) = 90^{\circ} 0' 0.09''$	$\sigma_3 = 75^{\circ} 34' 27.67''$
$\frac{1}{2} (X - A) = 14^{\circ} 32' 3.13''$	$\frac{1}{2} (Y - B) = 14^{\circ} 32' 3.13''$	$\frac{1}{2} (Z - C) = 14^{\circ} 32' 3.13''$	$\sigma_4 = 79^{\circ} 3' 24.35''$
$a + x = 8.5475$	$b + y = 8.7643$	$z + z = 9.8579$	$360^{\circ} 0' 0''$
$x - a = 1.4109$	$y - b = 1.4109$	$z - c = 1.4109$	

EXAMPLES. X.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}Y, \frac{1}{2}X, \frac{1}{2}Z$	VOLUME.	REMARKS.
1. $\begin{cases} a=5.4669, \\ b=5.6185, \\ c=5.6661, \end{cases}$	$\begin{cases} 43.37^{\circ}59.08' \\ 47.33^{\circ}7.24' \\ 47.34^{\circ}34.13' \end{cases}$	$\begin{cases} 28^{\circ}16'2.20'' \\ 21^{\circ}33'12.17'' \\ 21^{\circ}33'12.17'' \end{cases}$	$\begin{cases} 20.51^{\circ}14.07'' \\ 30.49^{\circ}33.47'' \\ 23^{\circ}9'0.95'' \end{cases}$	$\begin{cases} 20.53^{\circ}40.59'' \\ 32^{\circ}12.59.97'' \\ 30.54^{\circ}24.33'' \end{cases}$	$\begin{cases} 24^{\circ}26'10.46'' \\ 27^{\circ}29'34.59'' \\ 27^{\circ}28'3.80'' \end{cases}$	$\begin{cases} 43^{\circ}51'10.86'' \\ 47^{\circ}28'29.46'' \\ 47^{\circ}28'0.18'' \end{cases}$	28.44716	$S_1=\sigma_1=138^{\circ}45'40.44''$
2. $\begin{cases} a=7.4779, \\ b=8.1235, \\ c=8.4597, \end{cases}$	$\begin{cases} 19^{\circ}15'31.94'' \\ 17^{\circ}18'32.99'' \\ 16^{\circ}35'47.81'' \end{cases}$	$\begin{cases} 33^{\circ}33'48.79'' \\ 40^{\circ}39'13.97'' \\ 41^{\circ}55'14.18'' \end{cases}$	$\begin{cases} 32^{\circ}44'58.22'' \\ 28.27^{\circ}49.45'' \\ 37^{\circ}0'59.50'' \end{cases}$	$\begin{cases} 30^{\circ}46'12.83'' \\ 33.43^{\circ}28.50'' \\ 25^{\circ}38'21.76'' \end{cases}$	$\begin{cases} 35^{\circ}24'48.19'' \\ 41^{\circ}25'24.17'' \\ 43^{\circ}9'58.40'' \end{cases}$	$\begin{cases} 36^{\circ}13'44.26'' \\ 30^{\circ}13'8.23'' \\ 28^{\circ}28'34.01'' \end{cases}$	25.39085	
3. $\begin{cases} a=7.4779, \\ b=8.1235, \\ c=8.4597, \end{cases}$	$\begin{cases} 18^{\circ}1'21.39'' \\ 15^{\circ}11'40.71'' \\ 16^{\circ}35'47.81'' \end{cases}$	$\begin{cases} 35^{\circ}33'48.79'' \\ 47^{\circ}39'13.97'' \\ 47^{\circ}27'2.80'' \end{cases}$	$\begin{cases} 32^{\circ}44'58.22'' \\ 28.27^{\circ}49.45'' \\ 41^{\circ}55'14.18'' \end{cases}$	$\begin{cases} 26.51^{\circ}16.49'' \\ 30.22^{\circ}56.90'' \\ 25^{\circ}38'21.76'' \end{cases}$	$\begin{cases} 32^{\circ}33'45.21'' \\ 39^{\circ}35'7.60'' \\ 49^{\circ}3'38.80'' \end{cases}$	$\begin{cases} 35^{\circ}21'19.15'' \\ 28^{\circ}19'56.74'' \\ 30^{\circ}42'38.45'' \end{cases}$	25.18782	$S_1=\sigma_1=124^{\circ}0'55.55''$
4. $\begin{cases} a=3.3333, \\ b=3.1111, \\ c=4.668152, \end{cases}$	$\begin{cases} 29^{\circ}50'59.97'' \\ 19^{\circ}54'42.40'' \\ 29^{\circ}50'34.08'' \end{cases}$	$\begin{cases} 41^{\circ}18'47.80'' \\ 28^{\circ}18'50.73'' \\ 47^{\circ}24'30.55'' \end{cases}$	$\begin{cases} 31^{\circ}44'28.59'' \\ 23^{\circ}10'48.39'' \\ 39^{\circ}36'10.95'' \end{cases}$	$\begin{cases} 22^{\circ}40'47.07'' \\ 20^{\circ}20'49.08'' \\ 25^{\circ}30'23.75'' \end{cases}$	$\begin{cases} 31^{\circ}44'53.38'' \\ 27^{\circ}42'45.74'' \\ 50^{\circ}22'39.20'' \end{cases}$	$\begin{cases} 38^{\circ}56'50.18'' \\ 23^{\circ}11'2.35'' \\ 38^{\circ}55'41.93'' \end{cases}$	4.370334	$S_2=\sigma_2=117^{\circ}2'15.12''$
5. $\begin{cases} a=5.2669, \\ b=7.8712, \\ c=7.8836, \end{cases}$	$\begin{cases} 28^{\circ}18'47.07'' \\ 21^{\circ}36'9.84'' \\ 21^{\circ}35'7.92'' \end{cases}$	$\begin{cases} 45^{\circ}39'16.88'' \\ 47^{\circ}30'47.98'' \\ 47^{\circ}32'18.12'' \end{cases}$	$\begin{cases} 20^{\circ}54'4.10'' \\ 23^{\circ}12'25.39'' \\ 30^{\circ}53'7.14'' \end{cases}$	$\begin{cases} 20.51^{\circ}32.04'' \\ 30.48^{\circ}5.79'' \\ 23^{\circ}8'47.73'' \end{cases}$	$\begin{cases} 24^{\circ}27'40.25'' \\ 47^{\circ}24'21.59'' \\ 47^{\circ}25'55.78'' \end{cases}$	$\begin{cases} 43^{\circ}52'5.60'' \\ 27^{\circ}30'1.99'' \\ 27^{\circ}28'27.88'' \end{cases}$	25.49455	$S_2=\sigma_2=138^{\circ}42'22.97''$
6. $\begin{cases} a=2.6335, \\ b=2.7093, \\ c=3.8318, \end{cases}$	$\begin{cases} 44^{\circ}30'31.86'' \\ 45^{\circ}50'43.40'' \\ 45^{\circ}58'17.40'' \end{cases}$	$\begin{cases} 29^{\circ}37'2.01'' \\ 22^{\circ}24'25.69'' \\ 22^{\circ}23'19.59'' \end{cases}$	$\begin{cases} 21^{\circ}37'17.11'' \\ 30^{\circ}9'2.45'' \\ 22^{\circ}42'43.89'' \end{cases}$	$\begin{cases} 21^{\circ}39'56.95'' \\ 22^{\circ}46'44.25'' \\ 30^{\circ}13'55.51'' \end{cases}$	$\begin{cases} 26^{\circ}0'41.50'' \\ 27^{\circ}32'57.90'' \\ 27^{\circ}31'22.62'' \end{cases}$	$\begin{cases} 44^{\circ}32'25.39'' \\ 45^{\circ}55'45.88'' \\ 45^{\circ}57'21.25'' \end{cases}$	3.210399	$S_1=\sigma_1=136^{\circ}25'32.52''$
7. $\begin{cases} a=2.6155, \\ b=2.6093, \\ c=2.6031, \end{cases}$	$\begin{cases} 45^{\circ}24'13.12'' \\ 49^{\circ}43.68'' \\ 44^{\circ}11'18.05'' \end{cases}$	$\begin{cases} 30^{\circ}39'53.85'' \\ 22^{\circ}32'14.79'' \\ 22^{\circ}51'0.31'' \end{cases}$	$\begin{cases} 22^{\circ}56'26.50'' \\ 22^{\circ}13'34.91'' \\ 22^{\circ}13'52.49'' \end{cases}$	$\begin{cases} 22^{\circ}59'25.01'' \\ 22^{\circ}19'54.39'' \\ 29^{\circ}42'31.29'' \end{cases}$	$\begin{cases} 28^{\circ}18'19.42'' \\ 27^{\circ}21'8.69'' \\ 27^{\circ}19'26.23'' \end{cases}$	$\begin{cases} 45^{\circ}25'40.03'' \\ 44^{\circ}1'51.98'' \\ 44^{\circ}10'34.47'' \end{cases}$	2.958072	$S_1=\sigma_1=133^{\circ}45'6.48''$
8. $\begin{cases} a=4.4877, \\ b=5.4871, \\ c=6.6877, \end{cases}$	$\begin{cases} 32^{\circ}39'13.33'' \\ 23^{\circ}52'5.53'' \\ 25^{\circ}52'59.29'' \end{cases}$	$\begin{cases} 44^{\circ}56'27.89'' \\ 37^{\circ}53'42.32'' \\ 45^{\circ}3'29.68'' \end{cases}$	$\begin{cases} 26^{\circ}13'18.39'' \\ 24^{\circ}3'49.92'' \\ 33^{\circ}2'41.31'' \end{cases}$	$\begin{cases} 21^{\circ}4'24.79'' \\ 24^{\circ}18'5.36'' \\ 20^{\circ}59'42.19'' \end{cases}$	$\begin{cases} 30^{\circ}3'56.67'' \\ 37^{\circ}53'41.77'' \\ 45^{\circ}4'30.01'' \end{cases}$	$\begin{cases} 44^{\circ}55'28.11'' \\ 22^{\circ}16'14.54'' \\ 29^{\circ}54'54.77'' \end{cases}$	15.95682	$S_2=\sigma_2=127^{\circ}53'39.89''$
9. $\begin{cases} a=4.6739, \\ b=4.5887, \\ c=4.4435, \end{cases}$	$\begin{cases} 47^{\circ}11'6.93'' \\ 45^{\circ}5'19.44'' \\ 44^{\circ}15'14.00'' \end{cases}$	$\begin{cases} 30^{\circ}47'39.37'' \\ 22^{\circ}30'22.25'' \\ 21^{\circ}18'32.58'' \end{cases}$	$\begin{cases} 23^{\circ}14'23.60'' \\ 30^{\circ}1'43.41'' \\ 20^{\circ}40'25.06'' \end{cases}$	$\begin{cases} 22^{\circ}42'28.08'' \\ 21^{\circ}29'28.01'' \\ 29^{\circ}10'37.22'' \end{cases}$	$\begin{cases} 28^{\circ}22'50.84'' \\ 20^{\circ}35'44.75'' \\ 25^{\circ}1'14.87'' \end{cases}$	$\begin{cases} 47^{\circ}39'43.28'' \\ 45^{\circ}35'3.45'' \\ 44^{\circ}20'33.60'' \end{cases}$	15.66689	$S_1=\sigma_1=137^{\circ}55'20.33''$
10. $\begin{cases} a=8.7737, \\ b=7.8837, \\ c=7.7745, \end{cases}$	$\begin{cases} 30^{\circ}13'59.65'' \\ 31^{\circ}51'17.00'' \\ 31^{\circ}12'45.19'' \end{cases}$	$\begin{cases} 26^{\circ}52'34.55'' \\ 26^{\circ}52'26.72'' \\ 26^{\circ}17'50.37'' \end{cases}$	$\begin{cases} 31^{\circ}54'48.09'' \\ 31^{\circ}51'56.80'' \\ 29^{\circ}34'26.10'' \end{cases}$	$\begin{cases} 31^{\circ}50'51.73'' \\ 30^{\circ}13'34.25'' \\ 31^{\circ}12'28.65'' \end{cases}$	$\begin{cases} 36^{\circ}54'45.66'' \\ 34^{\circ}22'33.72'' \\ 33^{\circ}19'22.88'' \end{cases}$	$\begin{cases} 34^{\circ}22'48.48'' \\ 36^{\circ}55'0.43'' \\ 35^{\circ}51'49.59'' \end{cases}$	66.38926	$a+x=b+y, y=b+c$

EXAMPLES. N.—Continued.

DATA.	$\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	VOLUME.	REMARKS.
11. $\begin{cases} a=5.0837, \\ b=5.5789, \\ c=0.0741, \end{cases}$ $\begin{cases} A=9.9998 \\ B=9.5046 \\ C=9.0094 \end{cases}$	$\begin{cases} 59^\circ 4'34.92'' \\ 58^\circ 16'19.37'' \\ 57^\circ 37'39.92'' \end{cases}$	$\begin{cases} 32^\circ 39'35.66'' \\ 17^\circ 1'45.57'' \\ 17^\circ 26'4.64'' \end{cases}$	$\begin{cases} 15^\circ 20'34.51'' \\ 29^\circ 51'57.13'' \\ 16^\circ 11'27.03'' \end{cases}$	$\begin{cases} 14^\circ 17'35.99'' \\ 14^\circ 43'57.68'' \\ 27^\circ 28'31.21'' \end{cases}$	$\begin{cases} 10^\circ 3'55.74'' \\ 10^\circ 21'37.30'' \\ 71^\circ 55'11.05'' \end{cases}$	$\begin{cases} 72^\circ 29' 0.19'' \\ 71^\circ 57'44.74'' \\ 71^\circ 57'44.74'' \end{cases}$	13.33528	$a + x = b + y = c + z$
12. $\begin{cases} a=9.5667, \\ b=7.5449, \\ c=7.4337, \end{cases}$ $\begin{cases} A=9.8554 \\ B=7.4336 \\ C=7.3224 \end{cases}$	$\begin{cases} 41^\circ 8'35.07'' \\ 23^\circ 57' 6.10'' \\ 23^\circ 28'50.98'' \end{cases}$	$\begin{cases} 41^\circ 54' 9.67'' \\ 23^\circ 57' 7.63'' \\ 23^\circ 57' 7.63'' \end{cases}$	$\begin{cases} 42^\circ 5'20.70'' \\ 24^\circ 17'21.25'' \\ 24^\circ 11' 6.14'' \end{cases}$	$\begin{cases} 42^\circ 5'46.27'' \\ 24^\circ 18'18.79'' \\ 23^\circ 48'29.08'' \end{cases}$	$\begin{cases} 63^\circ 32'56.01'' \\ 18^\circ 43'48.04'' \\ 18^\circ 24'30.88'' \end{cases}$	$\begin{cases} 63^\circ 17'47.55'' \\ 18^\circ 28'40.16'' \\ 18^\circ 24'30.88'' \end{cases}$	40.56799	$a - A = b - B = c - C$
13. $\begin{cases} a=4.5567, \\ b=4.1233, \\ c=3.5903, \end{cases}$ $\begin{cases} A=5.5208 \\ B=5.9630 \\ C=6.14996 \end{cases}$	$\begin{cases} 44^\circ 57'34.70'' \\ 46^\circ 0'5.62'' \\ 45^\circ 2'31.61'' \end{cases}$	$\begin{cases} 27^\circ 8'57.92'' \\ 21^\circ 3' 9.00'' \\ 19^\circ 6'10.47'' \end{cases}$	$\begin{cases} 23^\circ 54'19.39'' \\ 30^\circ 33'47.88'' \\ 20^\circ 55'52.76'' \end{cases}$	$\begin{cases} 24^\circ 53'43.91'' \\ 34^\circ 6'32.54'' \\ 33^\circ 17'14.20'' \end{cases}$	$\begin{cases} 28^\circ 52' 4.34'' \\ 27^\circ 48'38.64'' \\ 27^\circ 48'12.50'' \end{cases}$	$\begin{cases} 44^\circ 57'39.91'' \\ 46^\circ 0'5.64'' \\ 45^\circ 2'31.61'' \end{cases}$	11.54943	$a + A = b + B = c + C$ $S_1 = \sigma = 136^\circ 0'11.94''$
14. $\begin{cases} a=2.3155, \\ b=3.421484, \\ c=3.7418, \end{cases}$ $\begin{cases} A=3.8118 \\ B=2.5093 \\ C=2.5033 \end{cases}$	$\begin{cases} 32^\circ 3'29.18'' \\ 20^\circ 34'11.20'' \\ 23^\circ 26' 8.63'' \end{cases}$	$\begin{cases} 49^\circ 30'11.85'' \\ 42^\circ 58'44.66'' \\ 48^\circ 7'49.49'' \end{cases}$	$\begin{cases} 23^\circ 35' 6.71'' \\ 20^\circ 16'39.50'' \\ 31^\circ 0'40.58'' \end{cases}$	$\begin{cases} 20^\circ 57'59.31'' \\ 26^\circ 55'44.24'' \\ 20^\circ 13' 8.05'' \end{cases}$	$\begin{cases} 27^\circ 51'33.87'' \\ 43^\circ 26'26.39'' \\ 47^\circ 51'17.81'' \end{cases}$	$\begin{cases} 49^\circ 19' 1.80'' \\ 23^\circ 33' 0.54'' \\ 26^\circ 39'39.91'' \end{cases}$	2.581718	$S_2 = \sigma = 140^\circ 36'46''$
15. $\begin{cases} a=7.8530, \\ b=6.4705, \\ c=6.6899, \end{cases}$ $\begin{cases} A=9.8528 \\ B=9.1375 \\ C=8.6897 \end{cases}$	$\begin{cases} 48^\circ 28' 5.76'' \\ 38^\circ 41'50.13'' \\ 36^\circ 59'44.95'' \end{cases}$	$\begin{cases} 33^\circ 31'26.50'' \\ 22^\circ 51' 7.29'' \\ 22^\circ 48' 2.45'' \end{cases}$	$\begin{cases} 30^\circ 9' 7.76'' \\ 29^\circ 19'25.24'' \\ 21^\circ 11'19.85'' \end{cases}$	$\begin{cases} 28^\circ 30' 7.42'' \\ 20^\circ 20'34.39'' \\ 27^\circ 9' 8.26'' \end{cases}$	$\begin{cases} 39^\circ 26'19.75'' \\ 24^\circ 51' 1.57'' \\ 24^\circ 47'28.95'' \end{cases}$	$\begin{cases} 50^\circ 33'40.25'' \\ 37^\circ 21'27.09'' \\ 35^\circ 54'49.45'' \end{cases}$	52.14756	$A + A = 180^\circ$, vol. a max. for variations of a, b, c so that $\Delta a - \Delta b = 0$
16. $\begin{cases} a=4.3719, \\ b=5.3726, \\ c=5.6464, \end{cases}$ $\begin{cases} A=3.2608 \\ B=4.2615 \\ C=4.5353 \end{cases}$	$\begin{cases} 17^\circ 9'20.93'' \\ 24^\circ 9'22.73'' \\ 27^\circ 9' 9.70'' \end{cases}$	$\begin{cases} 21^\circ 41'16.39'' \\ 37^\circ 5' 5.53'' \\ 40^\circ 50'23.35'' \end{cases}$	$\begin{cases} 35^\circ 45'44.77'' \\ 31^\circ 55' 6.90'' \\ 38^\circ 43'22.87'' \end{cases}$	$\begin{cases} 25^\circ 0'13.92'' \\ 34^\circ 7'16.20'' \\ 36^\circ 23'36.71'' \end{cases}$	$\begin{cases} 26^\circ 33'13.41'' \\ 37^\circ 51'38.08'' \\ 47^\circ 19'31.17'' \end{cases}$	$\begin{cases} 21^\circ 55'41.07'' \\ 33^\circ 12'35.74'' \\ 42^\circ 40'28.83'' \end{cases}$	9.274307	$C + Z = 180^\circ$, vol. a max. for variations of a, b, c such that $\Delta a - \Delta b = 0$
17. $\begin{cases} a=2.1995, \\ b=2.3207, \\ c=2.6116, \end{cases}$ $\begin{cases} A=3.4633 \\ B=3.5845 \\ C=3.8754 \end{cases}$	$\begin{cases} 44^\circ 29'59.11'' \\ 47^\circ 38'25.35'' \\ 59^\circ 0'29.20'' \end{cases}$	$\begin{cases} 27^\circ 34'43.45'' \\ 15^\circ 57'26.28'' \\ 23^\circ 13'13.00'' \end{cases}$	$\begin{cases} 15^\circ 2' 4.52'' \\ 29^\circ 43'28.81'' \\ 24^\circ 28' 2.76'' \end{cases}$	$\begin{cases} 18^\circ 48'21.62'' \\ 21^\circ 1'58.13'' \\ 33^\circ 20'43.71'' \end{cases}$	$\begin{cases} 17^\circ 37' 2.56'' \\ 25^\circ 2' 6.13'' \\ 30^\circ 57'58.05'' \end{cases}$	$\begin{cases} 46^\circ 1' 6.46'' \\ 48^\circ 6'10.02'' \\ 59^\circ 2' 1.95'' \end{cases}$	1.949617	$A + A = B + B = C + C$
18. $\begin{cases} a=4.8613, \\ b=4.9233, \\ c=5.5121, \end{cases}$ $\begin{cases} A=4.7940 \\ B=4.7320 \\ C=4.1432 \end{cases}$	$\begin{cases} 27^\circ 10'15.48'' \\ 26^\circ 55' 6.86'' \\ 25^\circ 2'58.41'' \end{cases}$	$\begin{cases} 32^\circ 29'48.09'' \\ 32^\circ 51'48.98'' \\ 35^\circ 3'45.64'' \end{cases}$	$\begin{cases} 32^\circ 5'12.61'' \\ 31^\circ 43'28.55'' \\ 34^\circ 33' 3.73'' \end{cases}$	$\begin{cases} 28^\circ 1' 7.50'' \\ 28^\circ 16'40.79'' \\ 25^\circ 46'43.36'' \end{cases}$	$\begin{cases} 35^\circ 21' 0.99'' \\ 35^\circ 51'30.43'' \\ 39^\circ 11'51.04'' \end{cases}$	$\begin{cases} 35^\circ 21'48.53'' \\ 34^\circ 51'19.07'' \\ 31^\circ 30'58.46'' \end{cases}$	12.85487	$A + A = B + B = C + C$
19. $\begin{cases} a=4.3723, \\ b=5.3723, \\ c=5.6463, \end{cases}$ $\begin{cases} A=3.2612 \\ B=4.2612 \\ C=4.5352 \end{cases}$	$\begin{cases} 17^\circ 9'30.81'' \\ 24^\circ 9'15.71'' \\ 27^\circ 9' 9.15'' \end{cases}$	$\begin{cases} 21^\circ 41'29.90'' \\ 37^\circ 4'51.85'' \\ 40^\circ 50'17.88'' \end{cases}$	$\begin{cases} 25^\circ 45'50.00'' \\ 31^\circ 55'45.45'' \\ 38^\circ 43'22.85'' \end{cases}$	$\begin{cases} 25^\circ 0'26.41'' \\ 28^\circ 16'40.79'' \\ 30^\circ 23'34.65'' \end{cases}$	$\begin{cases} 26^\circ 32'28.31'' \\ 37^\circ 51'25.48'' \\ 47^\circ 19'32.84'' \end{cases}$	$\begin{cases} 21^\circ 53'25.40'' \\ 33^\circ 12'27.57'' \\ 42^\circ 40'29.93'' \end{cases}$	9.275219	$A - A = B - B = C - C$
20. $\begin{cases} a=4.8624, \\ b=4.9233, \\ c=5.5121, \end{cases}$ $\begin{cases} A=4.7920 \\ B=4.7320 \\ C=4.1432 \end{cases}$	$\begin{cases} 27^\circ 9'50.81'' \\ 26^\circ 54'58.42'' \\ 25^\circ 2'47.88'' \end{cases}$	$\begin{cases} 32^\circ 29'17.51'' \\ 32^\circ 51'29.46'' \\ 35^\circ 3'28.59'' \end{cases}$	$\begin{cases} 32^\circ 5'12.66'' \\ 31^\circ 43'28.55'' \\ 34^\circ 33'19.38'' \end{cases}$	$\begin{cases} 28^\circ 1'32.99'' \\ 28^\circ 16'49.81'' \\ 25^\circ 46'55.14'' \end{cases}$	$\begin{cases} 33^\circ 21'24.70'' \\ 35^\circ 51'21.74'' \\ 39^\circ 11'49.88'' \end{cases}$	$\begin{cases} 33^\circ 21'24.70'' \\ 34^\circ 51'27.66'' \\ 31^\circ 30'59.52'' \end{cases}$	12.85487	(18) differs from this by $\Delta a = \text{corr.}$, $\Delta x = \text{corr.}$, and since $\Delta a = \Delta x$, Δx is a max., and $\Delta x = 0$.

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}A$	$\frac{1}{2}B$	$\frac{1}{2}C$	$\frac{1}{2}D$	$\frac{1}{2}E$	$\frac{1}{2}F$	$\frac{1}{2}G$	$\frac{1}{2}H$	$\frac{1}{2}I$	$\frac{1}{2}J$	$\frac{1}{2}K$	$\frac{1}{2}L$	$\frac{1}{2}M$	$\frac{1}{2}N$	$\frac{1}{2}O$	$\frac{1}{2}P$	$\frac{1}{2}Q$	$\frac{1}{2}R$	$\frac{1}{2}S$	$\frac{1}{2}T$	$\frac{1}{2}U$	$\frac{1}{2}V$	$\frac{1}{2}W$	$\frac{1}{2}X$	$\frac{1}{2}Y$	$\frac{1}{2}Z$	VOLUME.	REMARKS.
21. $\left\{ \begin{array}{l} a=2.3309, \\ b=7.2250, \\ c=4.1705 \end{array} \right.$	$\frac{1}{2}A=7.1186$	$\frac{1}{2}B=2.3309$	$\frac{1}{2}C=7.2250$	$\frac{1}{2}D=4.1705$	$\frac{1}{2}E=2.3309$	$\frac{1}{2}F=7.2250$	$\frac{1}{2}G=4.1705$	$\frac{1}{2}H=2.3309$	$\frac{1}{2}I=7.2250$	$\frac{1}{2}J=4.1705$	$\frac{1}{2}K=2.3309$	$\frac{1}{2}L=7.2250$	$\frac{1}{2}M=4.1705$	$\frac{1}{2}N=2.3309$	$\frac{1}{2}O=7.2250$	$\frac{1}{2}P=4.1705$	$\frac{1}{2}Q=2.3309$	$\frac{1}{2}R=7.2250$	$\frac{1}{2}S=4.1705$	$\frac{1}{2}T=2.3309$	$\frac{1}{2}U=7.2250$	$\frac{1}{2}V=4.1705$	$\frac{1}{2}W=2.3309$	$\frac{1}{2}X=7.2250$	$\frac{1}{2}Y=4.1705$	$\frac{1}{2}Z=2.3309$	5.53466	$A+X=B+Y=C+Z$
22. $\left\{ \begin{array}{l} a=7.1186, \\ b=4.2250, \\ c=4.1705 \end{array} \right.$	$\frac{1}{2}A=7.1186$	$\frac{1}{2}B=4.2250$	$\frac{1}{2}C=4.1705$	$\frac{1}{2}D=7.1186$	$\frac{1}{2}E=4.2250$	$\frac{1}{2}F=4.1705$	$\frac{1}{2}G=7.1186$	$\frac{1}{2}H=4.2250$	$\frac{1}{2}I=4.1705$	$\frac{1}{2}J=7.1186$	$\frac{1}{2}K=4.2250$	$\frac{1}{2}L=4.1705$	$\frac{1}{2}M=7.1186$	$\frac{1}{2}N=4.2250$	$\frac{1}{2}O=4.1705$	$\frac{1}{2}P=7.1186$	$\frac{1}{2}Q=4.2250$	$\frac{1}{2}R=4.1705$	$\frac{1}{2}S=7.1186$	$\frac{1}{2}T=4.2250$	$\frac{1}{2}U=4.1705$	$\frac{1}{2}V=7.1186$	$\frac{1}{2}W=4.2250$	$\frac{1}{2}X=4.1705$	$\frac{1}{2}Y=7.1186$	$\frac{1}{2}Z=4.2250$	7.857833	$A+X=B+Y=C+Z$
23. $\left\{ \begin{array}{l} a=7.1186, \\ b=4.2245, \\ c=4.1705 \end{array} \right.$	$\frac{1}{2}A=7.1186$	$\frac{1}{2}B=4.2245$	$\frac{1}{2}C=4.1705$	$\frac{1}{2}D=7.1186$	$\frac{1}{2}E=4.2245$	$\frac{1}{2}F=4.1705$	$\frac{1}{2}G=7.1186$	$\frac{1}{2}H=4.2245$	$\frac{1}{2}I=4.1705$	$\frac{1}{2}J=7.1186$	$\frac{1}{2}K=4.2245$	$\frac{1}{2}L=4.1705$	$\frac{1}{2}M=7.1186$	$\frac{1}{2}N=4.2245$	$\frac{1}{2}O=4.1705$	$\frac{1}{2}P=7.1186$	$\frac{1}{2}Q=4.2245$	$\frac{1}{2}R=4.1705$	$\frac{1}{2}S=7.1186$	$\frac{1}{2}T=4.2245$	$\frac{1}{2}U=4.1705$	$\frac{1}{2}V=7.1186$	$\frac{1}{2}W=4.2245$	$\frac{1}{2}X=4.1705$	$\frac{1}{2}Y=7.1186$	$\frac{1}{2}Z=4.2245$	7.857836	$A+X=B+Y=C+Z$
24. $\left\{ \begin{array}{l} a=7.1187, \\ b=4.2237, \\ c=4.1697 \end{array} \right.$	$\frac{1}{2}A=7.1187$	$\frac{1}{2}B=4.2237$	$\frac{1}{2}C=4.1697$	$\frac{1}{2}D=7.1187$	$\frac{1}{2}E=4.2237$	$\frac{1}{2}F=4.1697$	$\frac{1}{2}G=7.1187$	$\frac{1}{2}H=4.2237$	$\frac{1}{2}I=4.1697$	$\frac{1}{2}J=7.1187$	$\frac{1}{2}K=4.2237$	$\frac{1}{2}L=4.1697$	$\frac{1}{2}M=7.1187$	$\frac{1}{2}N=4.2237$	$\frac{1}{2}O=4.1697$	$\frac{1}{2}P=7.1187$	$\frac{1}{2}Q=4.2237$	$\frac{1}{2}R=4.1697$	$\frac{1}{2}S=7.1187$	$\frac{1}{2}T=4.2237$	$\frac{1}{2}U=4.1697$	$\frac{1}{2}V=7.1187$	$\frac{1}{2}W=4.2237$	$\frac{1}{2}X=4.1697$	$\frac{1}{2}Y=7.1187$	$\frac{1}{2}Z=4.2237$	7.857187	$A+X=B+Y=C+Z$
25. $\left\{ \begin{array}{l} a=8.8524, \\ b=7.6523, \\ c=8.6119 \end{array} \right.$	$\frac{1}{2}A=8.8524$	$\frac{1}{2}B=7.6523$	$\frac{1}{2}C=8.6119$	$\frac{1}{2}D=8.8524$	$\frac{1}{2}E=7.6523$	$\frac{1}{2}F=8.6119$	$\frac{1}{2}G=8.8524$	$\frac{1}{2}H=7.6523$	$\frac{1}{2}I=8.6119$	$\frac{1}{2}J=8.8524$	$\frac{1}{2}K=7.6523$	$\frac{1}{2}L=8.6119$	$\frac{1}{2}M=8.8524$	$\frac{1}{2}N=7.6523$	$\frac{1}{2}O=8.6119$	$\frac{1}{2}P=8.8524$	$\frac{1}{2}Q=7.6523$	$\frac{1}{2}R=8.6119$	$\frac{1}{2}S=8.8524$	$\frac{1}{2}T=7.6523$	$\frac{1}{2}U=8.6119$	$\frac{1}{2}V=8.8524$	$\frac{1}{2}W=7.6523$	$\frac{1}{2}X=8.6119$	$\frac{1}{2}Y=8.8524$	$\frac{1}{2}Z=7.6523$	69.37919	$C+Z=180^\circ$ very nearly
26. $\left\{ \begin{array}{l} a=8.8529, \\ b=7.6523, \\ c=8.6140 \end{array} \right.$	$\frac{1}{2}A=8.8529$	$\frac{1}{2}B=7.6523$	$\frac{1}{2}C=8.6140$	$\frac{1}{2}D=8.8529$	$\frac{1}{2}E=7.6523$	$\frac{1}{2}F=8.6140$	$\frac{1}{2}G=8.8529$	$\frac{1}{2}H=7.6523$	$\frac{1}{2}I=8.6140$	$\frac{1}{2}J=8.8529$	$\frac{1}{2}K=7.6523$	$\frac{1}{2}L=8.6140$	$\frac{1}{2}M=8.8529$	$\frac{1}{2}N=7.6523$	$\frac{1}{2}O=8.6140$	$\frac{1}{2}P=8.8529$	$\frac{1}{2}Q=7.6523$	$\frac{1}{2}R=8.6140$	$\frac{1}{2}S=8.8529$	$\frac{1}{2}T=7.6523$	$\frac{1}{2}U=8.6140$	$\frac{1}{2}V=8.8529$	$\frac{1}{2}W=7.6523$	$\frac{1}{2}X=8.6140$	$\frac{1}{2}Y=8.8529$	$\frac{1}{2}Z=7.6523$	69.37917	$A=X, B=Y$
27. $\left\{ \begin{array}{l} a=8.8529, \\ b=7.6514, \\ c=8.6119 \end{array} \right.$	$\frac{1}{2}A=8.8529$	$\frac{1}{2}B=7.6514$	$\frac{1}{2}C=8.6119$	$\frac{1}{2}D=8.8529$	$\frac{1}{2}E=7.6514$	$\frac{1}{2}F=8.6119$	$\frac{1}{2}G=8.8529$	$\frac{1}{2}H=7.6514$	$\frac{1}{2}I=8.6119$	$\frac{1}{2}J=8.8529$	$\frac{1}{2}K=7.6514$	$\frac{1}{2}L=8.6119$	$\frac{1}{2}M=8.8529$	$\frac{1}{2}N=7.6514$	$\frac{1}{2}O=8.6119$	$\frac{1}{2}P=8.8529$	$\frac{1}{2}Q=7.6514$	$\frac{1}{2}R=8.6119$	$\frac{1}{2}S=8.8529$	$\frac{1}{2}T=7.6514$	$\frac{1}{2}U=8.6119$	$\frac{1}{2}V=8.8529$	$\frac{1}{2}W=7.6514$	$\frac{1}{2}X=8.6119$	$\frac{1}{2}Y=8.8529$	$\frac{1}{2}Z=7.6514$	69.37920	
28. $\left\{ \begin{array}{l} a=8.8529, \\ b=7.6523, \\ c=8.6119 \end{array} \right.$	$\frac{1}{2}A=8.8529$	$\frac{1}{2}B=7.6523$	$\frac{1}{2}C=8.6119$	$\frac{1}{2}D=8.8529$	$\frac{1}{2}E=7.6523$	$\frac{1}{2}F=8.6119$	$\frac{1}{2}G=8.8529$	$\frac{1}{2}H=7.6523$	$\frac{1}{2}I=8.6119$	$\frac{1}{2}J=8.8529$	$\frac{1}{2}K=7.6523$	$\frac{1}{2}L=8.6119$	$\frac{1}{2}M=8.8529$	$\frac{1}{2}N=7.6523$	$\frac{1}{2}O=8.6119$	$\frac{1}{2}P=8.8529$	$\frac{1}{2}Q=7.6523$	$\frac{1}{2}R=8.6119$	$\frac{1}{2}S=8.8529$	$\frac{1}{2}T=7.6523$	$\frac{1}{2}U=8.6119$	$\frac{1}{2}V=8.8529$	$\frac{1}{2}W=7.6523$	$\frac{1}{2}X=8.6119$	$\frac{1}{2}Y=8.8529$	$\frac{1}{2}Z=7.6523$	69.37921	$A=X, B=Y, C=Z=180^\circ$. Y^a max. subject to $a+x=17.7088$, $b+y=15.3046$, $c-z=1.6228$
29. $\left\{ \begin{array}{l} a=7.1182, \\ b=4.2394, \\ c=4.1705 \end{array} \right.$	$\frac{1}{2}A=7.1182$	$\frac{1}{2}B=4.2394$	$\frac{1}{2}C=4.1705$	$\frac{1}{2}D=7.1182$	$\frac{1}{2}E=4.2394$	$\frac{1}{2}F=4.1705$	$\frac{1}{2}G=7.1182$	$\frac{1}{2}H=4.2394$	$\frac{1}{2}I=4.1705$	$\frac{1}{2}J=7.1182$	$\frac{1}{2}K=4.2394$	$\frac{1}{2}L=4.1705$	$\frac{1}{2}M=7.1182$	$\frac{1}{2}N=4.2394$	$\frac{1}{2}O=4.1705$	$\frac{1}{2}P=7.1182$	$\frac{1}{2}Q=4.2394$	$\frac{1}{2}R=4.1705$	$\frac{1}{2}S=7.1182$	$\frac{1}{2}T=4.2394$	$\frac{1}{2}U=4.1705$	$\frac{1}{2}V=7.1182$	$\frac{1}{2}W=4.2394$	$\frac{1}{2}X=4.1705$	$\frac{1}{2}Y=7.1182$	$\frac{1}{2}Z=4.2394$	7.859762	$A+X=B+Y=C+Z$
30. $\left\{ \begin{array}{l} a=3.570328, \\ b=3.6707, \\ c=4.2233 \end{array} \right.$	$\frac{1}{2}A=3.570328$	$\frac{1}{2}B=3.6707$	$\frac{1}{2}C=4.2233$	$\frac{1}{2}D=3.570328$	$\frac{1}{2}E=3.6707$	$\frac{1}{2}F=4.2233$	$\frac{1}{2}G=3.570328$	$\frac{1}{2}H=3.6707$	$\frac{1}{2}I=4.2233$	$\frac{1}{2}J=3.570328$	$\frac{1}{2}K=3.6707$	$\frac{1}{2}L=4.2233$	$\frac{1}{2}M=3.570328$	$\frac{1}{2}N=3.6707$	$\frac{1}{2}O=4.2233$	$\frac{1}{2}P=3.570328$	$\frac{1}{2}Q=3.6707$	$\frac{1}{2}R=4.2233$	$\frac{1}{2}S=3.570328$	$\frac{1}{2}T=3.6707$	$\frac{1}{2}U=4.2233$	$\frac{1}{2}V=3.570328$	$\frac{1}{2}W=3.6707$	$\frac{1}{2}X=4.2233$	$\frac{1}{2}Y=3.570328$	$\frac{1}{2}Z=3.6707$	8.638628	$A^m \cdot B^m \cdot C^m = 1^m - R = Z - C$

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$	VOLUME.	REMARKS.
31. $\left\{ \begin{array}{l} a=7.1182, \\ b=4.1092, \\ c=4.1697, \end{array} \right.$	$16^{\circ}21' 0.83''$ $24^{\circ} 7' 9.93''$ $23^{\circ} 38' 46.75''$	$12^{\circ}40' 35.66''$ $17^{\circ}26' 29.93''$ $17^{\circ}48' 37.61''$	$45^{\circ}54' 43.32''$ $39^{\circ}23' 18.62''$ $37^{\circ}32' 18.03''$	$48^{\circ} 4' 12.46''$ $36^{\circ} 6' 11.14''$ $37^{\circ}56' 10.72''$	$50^{\circ}40' 1.17''$ $34^{\circ}56' 52.18''$ $36^{\circ}30' 8.93''$	$22^{\circ}20' 16.69''$ $38^{\circ} 3' 25.68''$ $36^{\circ}30' 8.93''$	7.833902	$A+X=B+Y=C+Z$ $C=Z$ I^{max} for vari- ations of c , z , such that $\Delta C+\Delta Z=0$
32. $\left\{ \begin{array}{l} a=7.1182, \\ b=4.1092, \\ c=4.1717, \end{array} \right.$	$16^{\circ}20' 44.92''$ $24^{\circ} 7' 15.23''$ $23^{\circ} 37' 56.09''$	$12^{\circ}40' 43.43''$ $17^{\circ}26' 44.55''$ $17^{\circ}49' 16.47''$	$48^{\circ}55' 39.36''$ $39^{\circ}24' 36.07''$ $37^{\circ}33' 50.95''$	$48^{\circ} 3' 28.28''$ $36^{\circ} 5' 24.10''$ $37^{\circ}54' 40.50''$	$50^{\circ}40' 11.50''$ $34^{\circ}55' 26.08''$ $36^{\circ}31' 48.45''$	$22^{\circ}20' 9.30''$ $38^{\circ} 4' 54.74''$ $36^{\circ}28' 32.35''$	7.833893	As compared with (31) $\Delta X=.0002$, $\Delta Y=-.002$, and $\Delta I=-.000009$
33. $\left\{ \begin{array}{l} a=7.1182, \\ b=4.1092, \\ c=4.1697, \end{array} \right.$	$16^{\circ}20' 55.64''$ $24^{\circ} 7' 5.87''$ $23^{\circ} 38' 42.59''$	$12^{\circ}40' 31.69''$ $17^{\circ}26' 27.26''$ $17^{\circ}48' 35.05''$	$48^{\circ}54' 30.15''$ $39^{\circ}23' 15.81''$ $37^{\circ}32' 20.86''$	$48^{\circ} 4' 19.08''$ $36^{\circ} 6' 43.50''$ $37^{\circ}56' 12.46''$	$50^{\circ}40' 7.06''$ $34^{\circ}56' 53.02''$ $36^{\circ}30' 10.72''$	$22^{\circ}20' 12.47''$ $38^{\circ} 3' 26.53''$ $36^{\circ}30' 8.81''$	7.833233	As compared with (31) $\Delta X=.0002=-\Delta Y$, and $\Delta I=-.000069$
34. $\left\{ \begin{array}{l} a=7.1181, \\ b=4.1093, \\ c=4.1698, \end{array} \right.$	$16^{\circ}21' 1.98''$ $24^{\circ} 7' 9.75''$ $23^{\circ} 38' 46.63''$	$12^{\circ}40' 38.53''$ $17^{\circ}26' 32.99''$ $17^{\circ}48' 40.67''$	$48^{\circ}54' 40.38''$ $39^{\circ}23' 12.06''$ $37^{\circ}32' 17.38''$	$48^{\circ} 4' 9.58''$ $36^{\circ} 6' 40.64''$ $37^{\circ}56' 9.41''$	$50^{\circ}39' 58.69''$ $34^{\circ}56' 52.29''$ $36^{\circ}30' 8.71''$	$22^{\circ}20' 18.42''$ $38^{\circ} 3' 24.82''$ $36^{\circ}30' 8.40''$	7.834277	As compared with (31) $\Delta X=.0001=-\Delta Y$, $\Delta I=.0001=-\Delta Z$, and $\Delta I'=0.00375$
35. $\left\{ \begin{array}{l} a=3.5679, \\ b=3.6767, \\ c=3.7893, \end{array} \right.$	$42^{\circ} 5' 6.64''$ $24^{\circ} 4' 24.10''$ $24^{\circ} 8' 42.83''$	$59^{\circ}17' 30.80''$ $53^{\circ}25' 37.71''$ $53^{\circ}25' 37.71''$	$12^{\circ}20' 7.56''$ $15^{\circ}13' 46.96''$ $23^{\circ}16' 36.13''$	$12^{\circ}29' 58.19''$ $21^{\circ} 8' 17.25''$ $15^{\circ}28' 42.24''$	$14^{\circ}31' 55.25''$ $60^{\circ}27' 22.45''$ $60^{\circ}24' 33.25''$	$64^{\circ} 0' 6.71''$ $18^{\circ} 4' 39.51''$ $18^{\circ} 7' 28.71''$	4.800386	An ill-conditioned tetra- hedron. $A+X=B+Y$ $=C+Z=78^{\circ}32' 1.96''$
36. $\left\{ \begin{array}{l} a=3.5679, \\ b=3.6767, \\ c=3.7893, \end{array} \right.$	$41^{\circ}49' 5.28''$ $43^{\circ}43' 22.38''$ $45^{\circ}52' 1.78''$	$58^{\circ}56' 15.44''$ $22^{\circ}29' 1.67''$ $24^{\circ} 2' 22.84''$	$21^{\circ}38' 56.55''$ $29^{\circ}57' 50.95''$ $24^{\circ}34' 25.72''$	$22^{\circ}14' 11.81''$ $23^{\circ}36' 29.00''$ $31^{\circ} 5' 53.61''$	$26^{\circ} 9' 52.94''$ $28^{\circ} 0' 27.61''$ $30^{\circ}24' 22.18''$	$41^{\circ}46' 28.39''$ $43^{\circ}37' 3.06''$ $46^{\circ} 0' 57.63''$	8.220333	$X-A=1^{\circ}-B=Z-C$
37. $\left\{ \begin{array}{l} a=3.5683, \\ b=3.6767, \\ c=3.7893, \end{array} \right.$	$38^{\circ}54' 33.39''$ $40^{\circ}33' 15.50''$ $51^{\circ} 1' 33.44''$	$27^{\circ}31' 59.08''$ $19^{\circ}50' 1.62''$ $27^{\circ}31' 59.08''$	$10^{\circ} 8' 24.94''$ $28^{\circ}26' 19.20''$ $27^{\circ}59' 22.78''$	$21^{\circ}55' 45.42''$ $23^{\circ} 6' 3.83''$ $34^{\circ} 1' 11.72''$	$23^{\circ} 9' 55.50''$ $38^{\circ}59' 11.04''$ $52^{\circ}14' 32.82''$	$37^{\circ}43' 14.16''$ $38^{\circ}59' 11.04''$ $52^{\circ}14' 32.82''$	8.631324	$X-A=1^{\circ}-B=Z-C$ $C=X$
38. $\left\{ \begin{array}{l} a=4.9793, \\ b=5.0876, \\ c=5.3298, \end{array} \right.$	$13^{\circ}29' 34.11''$ $24^{\circ}16' 45.24''$ $23^{\circ}29' 58.05''$	$18^{\circ}21' 43.65''$ $44^{\circ}57' 5.85''$ $46^{\circ} 0' 16.09''$	$21^{\circ}3' 46.10''$ $40^{\circ} 0' 1.15''$ $42^{\circ}32' 18.45''$	$19^{\circ}42' 58.67''$ $33^{\circ}58' 7.44''$ $31^{\circ}38' 15.20''$	$21^{\circ}35' 53.86''$ $45^{\circ}34' 17.41''$ $46^{\circ}44' 43.42''$	$18^{\circ}20' 8.16''$ $40^{\circ}18' 31.71''$ $37^{\circ} 8' 5.70''$	15.51394	$A-X=B-Y$ $B+Y=C+Z$
39. $\left\{ \begin{array}{l} a=4.9793, \\ b=5.0876, \\ c=5.3298, \end{array} \right.$	$10^{\circ}13' 31.16''$ $19^{\circ}18' 6.84''$ $18^{\circ} 6' 11.31''$	$31^{\circ}18' 50.85''$ $36^{\circ}53' 30.14''$ $39^{\circ}50' 12.48''$	$35^{\circ} 0' 18.55''$ $33^{\circ} 6' 15.51''$ $39^{\circ}32' 49.87''$	$30^{\circ}11' 40.68''$ $31^{\circ}13' 38.97''$ $35^{\circ}34' 53.64''$	$36^{\circ}24' 19.54''$ $38^{\circ}29' 20.72''$ $43^{\circ}17' 25.22''$	$32^{\circ} 8' 55.00''$ $34^{\circ}13' 56.18''$ $29^{\circ}23' 51.68''$	8.12844	$A-X=B-Y$ $B+Y=C+Z$
40. $\left\{ \begin{array}{l} a=2.4897, \\ b=9.1439, \\ c=9.175, \end{array} \right.$	$5^{\circ}27' 4.27''$ $29^{\circ}50' 3.39''$ $29^{\circ}43' 23.37''$	$6^{\circ} 5' 47.64''$ $52^{\circ}39' 53.54''$ $52^{\circ}47' 20.72''$	$7^{\circ}36' 43.09''$ $46^{\circ}20' 13.26''$ $46^{\circ}39' 53.08''$	$7^{\circ}22' 35.59''$ $37^{\circ}53' 2.65''$ $37^{\circ}33' 59.10''$	$6^{\circ}10' 29.56''$ $45^{\circ}15' 52.66''$ $46^{\circ}25' 37.45''$	$7^{\circ}36' 55.66''$ $43^{\circ}58' 26.56''$ $42^{\circ}48' 41.77''$	5.816192	$A-X=B-Y$, $B+Y=C+Z$. A most ill-con- ditioned tetrahedron.

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	VOLUME.	REMARKS.
41. $\left\{ \begin{array}{l} a=4.3723, \\ b=5.3723, \\ c=5.6335, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	9.274728	$A - X = B - Y = C - Z$
42. $\left\{ \begin{array}{l} a=3.3003, \\ b=4.4803, \\ c=3.7209, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	6.474979	$A - X = B - Y = C - Z$
43. $\left\{ \begin{array}{l} a=4.8801, \\ b=4.9233, \\ c=5.5117, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	12.85546	$A + X = B + Y = C + Z$
44. $\left\{ \begin{array}{l} a=9.41135, \\ b=8.76369, \\ c=8.07325, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	38.61827	$B = 1 = 90^\circ$; I' a max. for variations in b, y
45. $\left\{ \begin{array}{l} a=6.4321, \\ b=5.5433, \\ c=5.3391, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	10 51411	$A + X = B + Y = C + Z$. I' a max. for variations of c, z , such that $\Delta c + \Delta z = 0$
46. $\left\{ \begin{array}{l} a=7.809322, \\ b=6.00328, \\ c=8.810394, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	49.16382	$\zeta = \zeta_1$, $A + X = B + Y = C + Z$. I' a max. for variations of c, z , such that $\Delta c + \Delta z = 0$
47. $\left\{ \begin{array}{l} a=7.060273, \\ b=7.375129, \\ c=7.315125, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	43.1705	$\sigma_1 = e_1 = \frac{\pi}{2}$; whence $\sigma_1 - \beta_1 = \alpha_1 - \beta_1$, $\sigma_2 - \beta_2 = \alpha_2 - \beta_2$, $\sigma_3 - \beta_3 = \alpha_3 - \beta_3$, and $\sigma_4 - \gamma_4 = \alpha_4 - \gamma_4$
48. $\left\{ \begin{array}{l} a=3.671925, \\ b=3.440879, \\ c=3.917612, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	5 21751	
49. $\left\{ \begin{array}{l} a=3.405534, \\ b=3.299272, \\ c=3.914063, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	4 825503	
50. $\left\{ \begin{array}{l} a=6.064341, \\ b=4.950051, \\ c=5.3391, \end{array} \right.$	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$	13.65486	$A + X = 180^\circ$, $B = 1$; I' a max. if $a - x = c - z = 0$, $b + y = c + z = 9.5981$

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$, $\frac{1}{2}A+B$, $\frac{1}{2}A+C$, $\frac{1}{2}B+C$, $\frac{1}{2}A+B+C$, $\frac{1}{2}A+B+C$, $\frac{1}{2}A+B+C$	VOLUME.	REMARKS.
51. $\begin{cases} a=9.869433, & x=5.619933 \\ b=8.180923, & y=5.522177 \\ c=8.9763, & z=4.7268 \end{cases}$	$\frac{1}{2}a=4.934715, \frac{1}{2}b=4.090461, \frac{1}{2}c=4.48815, \frac{1}{2}A=41^{\circ}21'36", \frac{1}{2}B=38^{\circ}43'35.12", \frac{1}{2}C=31^{\circ}12'11.5"$	31.12115	$A+B+C=180^{\circ}$ $B+C=180^{\circ}-A$ $A+C=180^{\circ}-B$ $A+B=180^{\circ}-C$
52. $\begin{cases} a=6.4141, & x=2.9120 \\ b=5.7819, & y=3.5442 \\ c=5.4637, & z=3.8624 \end{cases}$	$\frac{1}{2}a=3.20705, \frac{1}{2}b=2.89095, \frac{1}{2}c=2.93125, \frac{1}{2}A=44^{\circ}3'30.31", \frac{1}{2}B=31^{\circ}18'12.63", \frac{1}{2}C=26^{\circ}34'0.21"$	8.867272	$2a^2+x^2=b^2+y^2+c^2+z^2$ volume unaltered by interchange of y and z .
53. $\begin{cases} a=6.4141, & x=2.9120 \\ b=5.7819, & y=3.5442 \\ c=5.4637, & z=3.8624 \end{cases}$	$\frac{1}{2}a=3.20705, \frac{1}{2}b=2.89095, \frac{1}{2}c=2.93125, \frac{1}{2}A=44^{\circ}3'30.31", \frac{1}{2}B=31^{\circ}18'12.63", \frac{1}{2}C=26^{\circ}34'0.21"$	8.867270	Y ol. equal to that of A supplementary
54. $\begin{cases} a=6.4141, & x=2.9120 \\ b=5.7819, & y=3.5442 \\ c=5.4637, & z=3.8624 \end{cases}$	$\frac{1}{2}a=3.20705, \frac{1}{2}b=2.89095, \frac{1}{2}c=2.93125, \frac{1}{2}A=44^{\circ}3'30.31", \frac{1}{2}B=31^{\circ}18'12.63", \frac{1}{2}C=26^{\circ}34'0.21"$	2.455306	Very small angles involved in the calculation.
55. $\begin{cases} a=6.3571, & x=5.2896 \\ b=5.7073, & y=5.9394 \\ c=4.4077, & z=4.6398 \end{cases}$	$\frac{1}{2}a=3.17855, \frac{1}{2}b=2.85365, \frac{1}{2}c=2.20385, \frac{1}{2}A=37^{\circ}22'43.96", \frac{1}{2}B=30^{\circ}55'57.1", \frac{1}{2}C=27^{\circ}16'4.72"$	16.29656	$A+B+C=180^{\circ}$ $A+B=180^{\circ}-C$ $A+C=180^{\circ}-B$ $B+C=180^{\circ}-A$
56. $\begin{cases} a=6.3571, & x=5.2896 \\ b=5.7073, & y=5.9394 \\ c=4.4077, & z=4.6398 \end{cases}$	$\frac{1}{2}a=3.17855, \frac{1}{2}b=2.85365, \frac{1}{2}c=2.20385, \frac{1}{2}A=37^{\circ}22'43.96", \frac{1}{2}B=30^{\circ}55'57.1", \frac{1}{2}C=27^{\circ}16'4.72"$	16.29656	In (55), (56) the vols. are equal; the values of A supplementary.
57. $\begin{cases} a=7.4839, & x=3.5744 \\ b=7.2605, & y=3.3510 \\ c=6.8137, & z=3.3510 \end{cases}$	$\frac{1}{2}a=3.74195, \frac{1}{2}b=3.63025, \frac{1}{2}c=3.40685, \frac{1}{2}A=41^{\circ}27'25.54", \frac{1}{2}B=31^{\circ}52'48.08", \frac{1}{2}C=27^{\circ}16'4.72"$	12.3818	$A+B+C=180^{\circ}$ $A+B=180^{\circ}-C$ $A+C=180^{\circ}-B$ $B+C=180^{\circ}-A$
58. $\begin{cases} a=7.4839, & x=3.5744 \\ b=7.2605, & y=3.3510 \\ c=6.8137, & z=3.3510 \end{cases}$	$\frac{1}{2}a=3.74195, \frac{1}{2}b=3.63025, \frac{1}{2}c=3.40685, \frac{1}{2}A=41^{\circ}27'25.54", \frac{1}{2}B=31^{\circ}52'48.08", \frac{1}{2}C=27^{\circ}16'4.72"$	12.3818	Vols. of (57), (58) equal, and angle A supplementary.
59. $\begin{cases} a=7.1184, & x=2.3313 \\ b=4.1092, & y=5.3402 \\ c=4.1092, & z=5.2778 \end{cases}$	$\frac{1}{2}a=3.5592, \frac{1}{2}b=2.0546, \frac{1}{2}c=2.0546, \frac{1}{2}A=50^{\circ}40'18.81", \frac{1}{2}B=28^{\circ}20'12.36", \frac{1}{2}C=28^{\circ}20'12.36"$	7.8337865	Compared with (59) $\Delta I = -\cos \sigma$ $\Delta I' = -\cos \sigma$ $\Delta I'' = -\cos \sigma$ $\Delta I''' = -\cos \sigma$
60. $\begin{cases} a=7.7554, & x=6.9454 \\ b=5.32601, & y=6.51601 \\ c=5.70601, & z=5.33721 \end{cases}$	$\frac{1}{2}a=3.8777, \frac{1}{2}b=2.663005, \frac{1}{2}c=2.853005, \frac{1}{2}A=46^{\circ}38'49.39", \frac{1}{2}B=33^{\circ}40'38.27", \frac{1}{2}C=33^{\circ}40'38.27"$	29.33436	$A+B+C=180^{\circ}$ $A+B=180^{\circ}-C$ $A+C=180^{\circ}-B$ $B+C=180^{\circ}-A$

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}a_5, \frac{1}{2}b_5, \frac{1}{2}c_5$	$\frac{1}{2}a_6, \frac{1}{2}b_6, \frac{1}{2}c_6$	VOLUME.	REMARKS.
61. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 10^\circ 3'50.01'' \\ 20^\circ 25'47.66'' \\ 19^\circ 59'26.14'' \end{cases}$	$\begin{cases} 22^\circ 37' 3.57'' \\ 32^\circ 30'52.49'' \\ 22^\circ 23'55.02'' \end{cases}$	$\begin{cases} 37^\circ 23'41.37'' \\ 33^\circ 10'59.37'' \\ 23^\circ 49'31.45'' \end{cases}$	$\begin{cases} 47^\circ 10'17.32'' \\ 50^\circ 6'38.54'' \\ 49^\circ 38'31.91'' \end{cases}$	$\begin{cases} 45^\circ 19'31.89'' \\ 49^\circ 38'31.91'' \\ 24^\circ 59'26.10'' \end{cases}$	$\begin{cases} 25^\circ 15'13.49'' \\ 35^\circ 50'26.60'' \\ 34^\circ 24'16.94'' \end{cases}$	13.09054	This group of ten tetrahedra, (61) to (70), have all the same six lengths for their edges; and if these, in order of descending magnitude, be denoted by l, m, n, p, q, r , the following relations will hold
62. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 16^\circ 3'50.01'' \\ 20^\circ 49'36.58'' \\ 19^\circ 40'57.73'' \end{cases}$	$\begin{cases} 22^\circ 37' 3.57'' \\ 32^\circ 43'41.30'' \\ 22^\circ 25'39.39'' \end{cases}$	$\begin{cases} 37^\circ 35'20.97'' \\ 34^\circ 11'57.66'' \\ 23^\circ 49'31.45'' \end{cases}$	$\begin{cases} 46^\circ 11'44.03'' \\ 50^\circ 6'38.54'' \\ 33^\circ 10'59.37'' \end{cases}$	$\begin{cases} 44^\circ 40'28.11'' \\ 49^\circ 49'12.08'' \\ 24^\circ 55'43.10'' \end{cases}$	$\begin{cases} 25^\circ 15'13.49'' \\ 37^\circ 24'46.41'' \\ 33^\circ 18' 4.32'' \end{cases}$	13.69054	$3^2 + 2^2 = 3m^2 + 2q^2$ $= r^2 + m^2 + n^2 + p^2$ Denoting these tetrahedra by the suffixes 1 to 10, it will be seen that
63. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 22^\circ 28' 7.17'' \\ 20^\circ 35'47.66'' \\ 15^\circ 5'20.86'' \end{cases}$	$\begin{cases} 34^\circ 11'57.06'' \\ 34^\circ 14'11.92'' \\ 22^\circ 23'55.02'' \end{cases}$	$\begin{cases} 40^\circ 10'37.22'' \\ 33^\circ 10'59.37'' \\ 23^\circ 49'31.45'' \end{cases}$	$\begin{cases} 47^\circ 10'17.32'' \\ 43^\circ 15'15.97'' \\ 23^\circ 37' 3.57'' \end{cases}$	$\begin{cases} 49^\circ 49' 2.08'' \\ 40^\circ 21'25.01'' \\ 24^\circ 55'43.10'' \end{cases}$	$\begin{cases} 40^\circ 16' 5.13'' \\ 33^\circ 0'14.13'' \\ 23^\circ 17'38.76'' \end{cases}$	13.69054	l, m, n, p, q, r , the following relations will hold
64. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 15^\circ 40'18.85'' \\ 20^\circ 14'28.81'' \\ 21^\circ 16'55.00'' \end{cases}$	$\begin{cases} 21^\circ 48'38.61'' \\ 32^\circ 7'34.10'' \\ 20^\circ 36'26.56'' \end{cases}$	$\begin{cases} 36^\circ 35'30.81'' \\ 31^\circ 26'51.14'' \\ 21^\circ 28' 4.14'' \end{cases}$	$\begin{cases} 49^\circ 9' 4.63'' \\ 53^\circ 51'37.01'' \\ 36^\circ 44'30.25'' \end{cases}$	$\begin{cases} 47^\circ 5'35.70'' \\ 55^\circ 0'43.10'' \\ 22^\circ 36'20.02'' \end{cases}$	$\begin{cases} 23^\circ 59'54.11'' \\ 34^\circ 15'15.73'' \\ 37^\circ 33'32.38'' \end{cases}$	12.977715	$3^2 + 2^2 = 3m^2 + 2q^2$ $= r^2 + m^2 + n^2 + p^2$ Denoting these tetrahedra by the suffixes 1 to 10, it will be seen that
65. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 15^\circ 40'18.85'' \\ 22^\circ 25'39.30'' \\ 19^\circ 40'57.73'' \end{cases}$	$\begin{cases} 21^\circ 48'38.61'' \\ 32^\circ 43'41.30'' \\ 20^\circ 49'36.58'' \end{cases}$	$\begin{cases} 37^\circ 35'30.97'' \\ 36^\circ 44'30.25'' \\ 21^\circ 28' 4.14'' \end{cases}$	$\begin{cases} 46^\circ 44'44.03'' \\ 52^\circ 30'33.70'' \\ 31^\circ 26'51.14'' \end{cases}$	$\begin{cases} 42^\circ 54'24.30'' \\ 52^\circ 30'33.70'' \\ 22^\circ 19'25.23'' \end{cases}$	$\begin{cases} 23^\circ 25'13.22'' \\ 40^\circ 45'31.82'' \\ 31^\circ 24' 7.35'' \end{cases}$	12.977715	l, m, n, p, q, r , the following relations will hold
66. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 24^\circ 16'36.86'' \\ 20^\circ 14'28.81'' \\ 15^\circ 5'20.86'' \end{cases}$	$\begin{cases} 36^\circ 44'30.25'' \\ 34^\circ 14'11.92'' \\ 20^\circ 36'26.56'' \end{cases}$	$\begin{cases} 40^\circ 10'37.22'' \\ 31^\circ 26'51.14'' \\ 22^\circ 28' 7.17'' \end{cases}$	$\begin{cases} 49^\circ 9' 4.63'' \\ 43^\circ 15'15.97'' \\ 21^\circ 48'38.61'' \end{cases}$	$\begin{cases} 52^\circ 56'33.76'' \\ 37^\circ 59'16.84'' \\ 21^\circ 42'12.81'' \end{cases}$	$\begin{cases} 44^\circ 36'48.96'' \\ 30^\circ 0'15.61'' \\ 21^\circ 3'20.27'' \end{cases}$	12.97772	$3^2 + 2^2 = 3m^2 + 2q^2$ $= r^2 + m^2 + n^2 + p^2$ Denoting these tetrahedra by the suffixes 1 to 10, it will be seen that
67. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 15^\circ 32'40.76'' \\ 22^\circ 23'55.02'' \\ 19^\circ 59'26.14'' \end{cases}$	$\begin{cases} 21^\circ 40'53.76'' \\ 32^\circ 35'52.40'' \\ 20^\circ 25'47.66'' \end{cases}$	$\begin{cases} 37^\circ 23'41.37'' \\ 31^\circ 17'43.87'' \\ 20^\circ 53'38.77'' \end{cases}$	$\begin{cases} 47^\circ 10'17.32'' \\ 53^\circ 33'31.47'' \\ 32^\circ 122.37'' \end{cases}$	$\begin{cases} 43^\circ 13'41.35'' \\ 53^\circ 26' 7.04'' \\ 21^\circ 47' 7.55'' \end{cases}$	$\begin{cases} 23^\circ 10' 4.21'' \\ 40^\circ 21'56.63'' \\ 32^\circ 44'7.03'' \end{cases}$	12.83348	l, m, n, p, q, r , the following relations will hold
68. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 15^\circ 32'40.76'' \\ 20^\circ 36'26.56'' \\ 21^\circ 16'55.00'' \end{cases}$	$\begin{cases} 21^\circ 40'53.76'' \\ 32^\circ 7'34.10'' \\ 20^\circ 14'28.81'' \end{cases}$	$\begin{cases} 37^\circ 23'41.37'' \\ 30^\circ 35'30.81'' \\ 20^\circ 53'38.77'' \end{cases}$	$\begin{cases} 49^\circ 9' 4.63'' \\ 53^\circ 33'31.47'' \\ 36^\circ 17'43.87'' \end{cases}$	$\begin{cases} 46^\circ 46'15.65'' \\ 52^\circ 42'44.10'' \\ 22^\circ 0'22.90'' \end{cases}$	$\begin{cases} 23^\circ 38'49.86'' \\ 34^\circ 49' 9.49'' \\ 37^\circ 7'31.06'' \end{cases}$	12.83348	$3^2 + 2^2 = 3m^2 + 2q^2$ $= r^2 + m^2 + n^2 + p^2$ Denoting these tetrahedra by the suffixes 1 to 10, it will be seen that
69. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 23^\circ 49'31.44'' \\ 17^\circ 44'30.19'' \\ 19^\circ 59'26.14'' \end{cases}$	$\begin{cases} 32^\circ 18'35.14'' \\ 32^\circ 7'34.10'' \\ 13^\circ 22' 3.20'' \end{cases}$	$\begin{cases} 37^\circ 23'41.37'' \\ 28^\circ 30'37.55'' \\ 16^\circ 3'50.02'' \end{cases}$	$\begin{cases} 58^\circ 53'26.61'' \\ 50^\circ 6'38.54'' \\ 29^\circ 10'47.31'' \end{cases}$	$\begin{cases} 55^\circ 13' 5.36'' \\ 23^\circ 25'28.52'' \\ 14^\circ 47' 5.83'' \end{cases}$	$\begin{cases} 41^\circ 10'16.67'' \\ 25^\circ 33'48.52'' \\ 29^\circ 45' 4.60'' \end{cases}$	11.22593	l, m, n, p, q, r , the following relations will hold
70. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{cases}$	$\begin{cases} 21^\circ 28' 4.14'' \\ 17^\circ 44'30.19'' \\ 21^\circ 16'55.00'' \end{cases}$	$\begin{cases} 20^\circ 10'47.31'' \\ 32^\circ 7'34.10'' \\ 13^\circ 22' 3.20'' \end{cases}$	$\begin{cases} 36^\circ 35'30.81'' \\ 28^\circ 30'37.55'' \\ 15^\circ 40'18.85'' \end{cases}$	$\begin{cases} 58^\circ 53'26.61'' \\ 52^\circ 51'37.01'' \\ 32^\circ 18'35.14'' \end{cases}$	$\begin{cases} 56^\circ 51'51.45'' \\ 47^\circ 33'13.72'' \\ 14^\circ 54'59.08'' \end{cases}$	$\begin{cases} 35^\circ 8'53.00'' \\ 26^\circ 40'23.45'' \\ 34^\circ 36'10.81'' \end{cases}$	11.22593	$3^2 + 2^2 = 3m^2 + 2q^2$ $= r^2 + m^2 + n^2 + p^2$ Denoting these tetrahedra by the suffixes 1 to 10, it will be seen that

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	VOLUME.	REMARKS.
71. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 22^\circ 4'19.49'' \\ 20^\circ 49'36.58'' \\ 15^\circ 5'20.86'' \end{cases}$	$\begin{cases} 33^\circ 10'59.37'' \\ 22^\circ 49'36.58'' \\ 22^\circ 25'39.39'' \end{cases}$	$\begin{cases} 40^\circ 40'37.22'' \\ 34^\circ 14'11.92'' \\ 24^\circ 17'59.87'' \end{cases}$	$\begin{cases} 46^\circ 44'44.03'' \\ 43^\circ 37'40.61'' \\ 22^\circ 37'3.57'' \end{cases}$	$\begin{cases} 48^\circ 49'59.22'' \\ 41^\circ 19'16.21'' \\ 24^\circ 22'49.11'' \end{cases}$	$\begin{cases} 38^\circ 48'19.55'' \\ 13.77206 \\ 22^\circ 24'48.43'' \end{cases}$	
72. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 22^\circ 4'19.49'' \\ 14^\circ 18'6.32'' \\ 19^\circ 59'26.14'' \end{cases}$	$\begin{cases} 33^\circ 10'59.37'' \\ 32^\circ 36'52.49'' \\ 20^\circ 51'11.06'' \end{cases}$	$\begin{cases} 37^\circ 23'11.37'' \\ 22^\circ 37'3.57'' \\ 24^\circ 17'59.87'' \end{cases}$	$\begin{cases} 54^\circ 50'42.62'' \\ 43^\circ 37'40.61'' \\ 34^\circ 11'57.00'' \end{cases}$	$\begin{cases} 56^\circ 13'19.51'' \\ 39^\circ 20'13.50'' \\ 22^\circ 37'49.36'' \end{cases}$	$\begin{cases} 40^\circ 48'28.45'' \\ 21^\circ 25'25.05'' \\ 32^\circ 56'27.75'' \end{cases}$	12.84365
73. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 22^\circ 6'5.66'' \\ 22^\circ 35'39.31'' \\ 15^\circ 5'20.86'' \end{cases}$	$\begin{cases} 31^\circ 26'51.14'' \\ 34^\circ 14'11.02'' \\ 20^\circ 49'36.58'' \end{cases}$	$\begin{cases} 40^\circ 40'37.22'' \\ 36^\circ 41'30.25'' \\ 22^\circ 31'9.65'' \end{cases}$	$\begin{cases} 46^\circ 44'44.03'' \\ 45^\circ 22'11.69'' \\ 21^\circ 48'38.61'' \end{cases}$	$\begin{cases} 47^\circ 7'58.93'' \\ 43^\circ 42'58.08'' \\ 22^\circ 37'49.36'' \end{cases}$	$\begin{cases} 36^\circ 27'59.09'' \\ 37^\circ 37'46.06'' \\ 21^\circ 47'8.56'' \end{cases}$	13.47945
74. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 22^\circ 6'5.66'' \\ 13^\circ 34'58.03'' \\ 21^\circ 16'55.00'' \end{cases}$	$\begin{cases} 31^\circ 26'51.14'' \\ 32^\circ 7'34.19'' \\ 18^\circ 21'32.60'' \end{cases}$	$\begin{cases} 36^\circ 35'30.81'' \\ 31^\circ 48'38.61'' \\ 22^\circ 31'9.65'' \end{cases}$	$\begin{cases} 58^\circ 3'29.31'' \\ 45^\circ 22'44.69'' \\ 30^\circ 44'30.35'' \end{cases}$	$\begin{cases} 58^\circ 32'26.51'' \\ 41^\circ 15'54.61'' \\ 20^\circ 34'45.94'' \end{cases}$	$\begin{cases} 39^\circ 14'51.79'' \\ 10^\circ 57'32.53'' \\ 35^\circ 51'56.23'' \end{cases}$	12.03492
75. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 20^\circ 33'58.77'' \\ 20^\circ 51'11.06'' \\ 19^\circ 59'26.14'' \end{cases}$	$\begin{cases} 28^\circ 30'37.55'' \\ 32^\circ 36'52.49'' \\ 14^\circ 18'6.32'' \end{cases}$	$\begin{cases} 37^\circ 23'11.37'' \\ 32^\circ 18'35.14'' \\ 15^\circ 32'49.76'' \end{cases}$	$\begin{cases} 54^\circ 50'42.62'' \\ 53^\circ 33'31.47'' \\ 29^\circ 10'47.31'' \end{cases}$	$\begin{cases} 51^\circ 28'55.49'' \\ 49^\circ 13'2.84'' \\ 15^\circ 43'5.90'' \end{cases}$	$\begin{cases} 33^\circ 1'24.98'' \\ 32^\circ 55'14.21'' \\ 30^\circ 53'33.46'' \end{cases}$	11.86436
76. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 20^\circ 33'58.77'' \\ 18^\circ 21'32.66'' \\ 21^\circ 16'55.00'' \end{cases}$	$\begin{cases} 28^\circ 30'37.55'' \\ 32^\circ 36'52.49'' \\ 13^\circ 34'58.03'' \end{cases}$	$\begin{cases} 36^\circ 35'30.81'' \\ 29^\circ 10'47.31'' \\ 15^\circ 32'49.76'' \end{cases}$	$\begin{cases} 58^\circ 3'29.31'' \\ 53^\circ 33'31.47'' \\ 33^\circ 18'35.14'' \end{cases}$	$\begin{cases} 55^\circ 39'49.09'' \\ 48^\circ 44'29.8'' \\ 15^\circ 5'6.60'' \end{cases}$	$\begin{cases} 33^\circ 42'51.86'' \\ 27^\circ 58'17.7'' \\ 34^\circ 48'7.14'' \end{cases}$	11.34094
77. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 22^\circ 28'7.17'' \\ 19^\circ 40'57.73'' \\ 14^\circ 18'6.32'' \end{cases}$	$\begin{cases} 34^\circ 11'57.06'' \\ 32^\circ 43'11.30'' \\ 20^\circ 51'11.06'' \end{cases}$	$\begin{cases} 37^\circ 35'20.97'' \\ 22^\circ 37'3.57'' \\ 24^\circ 16'36.86'' \end{cases}$	$\begin{cases} 54^\circ 50'42.62'' \\ 43^\circ 15'15.97'' \\ 33^\circ 10'59.37'' \end{cases}$	$\begin{cases} 56^\circ 38'17.11'' \\ 38^\circ 26'2.57'' \\ 22^\circ 42'35.79'' \end{cases}$	$\begin{cases} 42^\circ 14'54.68'' \\ 21^\circ 12'50.88'' \\ 31^\circ 40'45.14'' \end{cases}$	12.7562
78. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 22^\circ 31'9.61'' \\ 22^\circ 23'55.02'' \\ 15^\circ 5'20.86'' \end{cases}$	$\begin{cases} 32^\circ 1'22.3'' \\ 34^\circ 14'11.02'' \\ 20^\circ 25'47.66'' \end{cases}$	$\begin{cases} 40^\circ 40'37.22'' \\ 36^\circ 17'43.87'' \\ 22^\circ 6'5.66'' \end{cases}$	$\begin{cases} 47^\circ 10'17.32'' \\ 45^\circ 22'44.69'' \\ 21^\circ 40'53.76'' \end{cases}$	$\begin{cases} 47^\circ 45'45.65'' \\ 43^\circ 14'56.18'' \\ 22^\circ 17'30.93'' \end{cases}$	$\begin{cases} 37^\circ 22'19.32'' \\ 36^\circ 56'27.33'' \\ 21^\circ 38'0.51'' \end{cases}$	13.39033
79. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 22^\circ 31'9.61'' \\ 13^\circ 22'3.20'' \\ 21^\circ 16'55.00'' \end{cases}$	$\begin{cases} 32^\circ 1'22.3'' \\ 32^\circ 7'34.19'' \\ 17^\circ 44'30.19'' \end{cases}$	$\begin{cases} 36^\circ 35'30.81'' \\ 21^\circ 40'53.76'' \\ 22^\circ 6'5.66'' \end{cases}$	$\begin{cases} 58^\circ 3'29.31'' \\ 45^\circ 22'44.69'' \\ 30^\circ 44'30.35'' \end{cases}$	$\begin{cases} 50^\circ 39'8.08'' \\ 40^\circ 12'2.21'' \\ 16^\circ 47'31.02'' \end{cases}$	$\begin{cases} 40^\circ 22'54.14'' \\ 19^\circ 26'3.19'' \\ 35^\circ 19'41.99'' \end{cases}$	11.7694
80. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=5.1220. \end{cases}$	$\begin{cases} 21^\circ 28'4.14'' \\ 20^\circ 51'11.06'' \\ 19^\circ 40'57.73'' \end{cases}$	$\begin{cases} 29^\circ 10'47.31'' \\ 32^\circ 43'11.30'' \\ 14^\circ 18'6.32'' \end{cases}$	$\begin{cases} 37^\circ 35'20.97'' \\ 32^\circ 18'35.14'' \\ 15^\circ 40'18.85'' \end{cases}$	$\begin{cases} 54^\circ 50'42.62'' \\ 52^\circ 51'37.01'' \\ 28^\circ 30'37.55'' \end{cases}$	$\begin{cases} 51^\circ 47'4.39'' \\ 48^\circ 10'54.20'' \\ 15^\circ 46'7.56'' \end{cases}$	$\begin{cases} 34^\circ 13'46.68'' \\ 32^\circ 38'55.92'' \\ 30^\circ 0'14.77'' \end{cases}$	11.9202

(71) to (84) are also tetrahedra with the same six lengths for their edges. The six tetrahedra in which the two longest edges are opposite each other are all impossible.

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1$, $\frac{1}{2}b_1$, $\frac{1}{2}c_1$	$\frac{1}{2}a_2$, $\frac{1}{2}b_2$, $\frac{1}{2}c_2$	$\frac{1}{2}a_3$, $\frac{1}{2}b_3$, $\frac{1}{2}c_3$	$\frac{1}{2}a_4$, $\frac{1}{2}b_4$, $\frac{1}{2}c_4$	$\frac{1}{2}a_5$, $\frac{1}{2}b_5$, $\frac{1}{2}c_5$	VOLUME.	REMARKS.
81. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}16'36.86'' \\ 13^{\circ}34'38.03'' \\ 19^{\circ}40'57.73'' \end{cases}$	$\begin{cases} 36^{\circ}44'30.25'' \\ 32^{\circ}43'41.30'' \\ 18^{\circ}21'32.00'' \end{cases}$	$\begin{cases} 37^{\circ}35'20.97'' \\ 22^{\circ}28'38.61'' \\ 21^{\circ}28'7.17'' \end{cases}$	$\begin{cases} 58^{\circ}3'29.31'' \\ 35^{\circ}27'15.97'' \\ 31^{\circ}20'51.14'' \end{cases}$	$\begin{cases} 60^{\circ}53'22.78'' \\ 35^{\circ}27'11.69'' \\ 19^{\circ}11'54.74'' \end{cases}$	$\begin{cases} 47^{\circ}34'3.05'' \\ 18^{\circ}40'55.74'' \\ 28^{\circ}43'0.00'' \end{cases}$	11.47017
82. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084
83. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084
84. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084
85. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084
86. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084
87. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084
88. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084
89. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084
90. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$	$\begin{cases} 24^{\circ}17'59.87'' \\ 13^{\circ}22'3.20'' \\ 19^{\circ}59'26.14'' \end{cases}$	$\begin{cases} 36^{\circ}17'43.87'' \\ 32^{\circ}30'52.49'' \\ 17^{\circ}44'30.19'' \end{cases}$	$\begin{cases} 37^{\circ}34'41.37'' \\ 21^{\circ}40'53.73'' \\ 23^{\circ}4'19.49'' \end{cases}$	$\begin{cases} 58^{\circ}53'26.61'' \\ 43^{\circ}37'40.64'' \\ 32^{\circ}1'22.37'' \end{cases}$	$\begin{cases} 61^{\circ}28'32.90'' \\ 35^{\circ}46'15.30'' \\ 18^{\circ}39'53.35'' \end{cases}$	$\begin{cases} 47^{\circ}17'4.47'' \\ 18^{\circ}21'26.04'' \\ 29^{\circ}19'2.54'' \end{cases}$	11.29084

See previous page.

These six, (86) to (91), are those, out of the thirty which have the same six lengths for their edges, in which the two longest edges are opposite. They are all "ill-conditioned," and their volume is much less than in any similar group of six.

EXAMPLES. N.—Continued.

	DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$	$\frac{1}{2}V, \frac{1}{2}Z$	VOLUME.	REMARKS.
91.	$\begin{cases} a=7.2485, \\ b=5.1220, \\ c=5.8856. \end{cases}$ $\begin{cases} x=6.8205, \\ y=4.7557, \\ z=4.8205. \end{cases}$	$\begin{cases} 40^{\circ}27' 9.94", \\ 20^{\circ}29' 50.39", \\ 20^{\circ}49' 36.58". \end{cases}$	$\begin{cases} 45^{\circ}22' 44.69", \\ 22^{\circ}25' 39.39", \\ 23^{\circ}59' 18.49". \end{cases}$	$\begin{cases} 40^{\circ}44' 44.03", \\ 22^{\circ}6' 5.06", \\ 23^{\circ}36' 58.45". \end{cases}$	$\begin{cases} 79^{\circ}30' 21.87", \\ 7^{\circ}27' 45.82", \\ 7^{\circ}52' 12.49". \end{cases}$	$\begin{cases} 79^{\circ}17' 30.88", \\ 7^{\circ}1' 9.65", \\ 7^{\circ}6' 49.33". \end{cases}$	5.305218	See previous page.
92.	$\begin{cases} a=7.3871, \\ b=5.9870, \\ c=6.1635. \end{cases}$ $\begin{cases} x=6.9870, \\ y=5.9634, \\ z=5.7634. \end{cases}$	$\begin{cases} 35^{\circ}10' 9.55", \\ 25^{\circ}37' 31.80", \\ 24^{\circ}52' 47.74". \end{cases}$	$\begin{cases} 36^{\circ}33' 34.19", \\ 26^{\circ}5' 8.05", \\ 20^{\circ}51' 23.75". \end{cases}$	$\begin{cases} 39^{\circ}2' 4.21", \\ 27^{\circ}22' 44.36", \\ 28^{\circ}5' 8.06". \end{cases}$	$\begin{cases} 37^{\circ}31' 4.45", \\ 26^{\circ}44' 11.79", \\ 26^{\circ}3' 41.45". \end{cases}$	$\begin{cases} 48^{\circ}16' 23.66", \\ 27^{\circ}40' 55.35", \\ 26^{\circ}49' 13.82". \end{cases}$	26.76496	
93.	$\begin{cases} a=7.1471, \\ b=5.9669, \\ c=6.1035. \end{cases}$ $\begin{cases} x=6.7470, \\ y=5.9600, \\ z=5.7634. \end{cases}$	$\begin{cases} 33^{\circ}46' 37.23", \\ 20^{\circ}17' 5.52", \\ 25^{\circ}35' 23.03". \end{cases}$	$\begin{cases} 35^{\circ}7' 27.48", \\ 20^{\circ}53' 1.12", \\ 27^{\circ}36' 10.53". \end{cases}$	$\begin{cases} 37^{\circ}31' 35.55", \\ 28^{\circ}7' 14.44", \\ 28^{\circ}45' 9.39". \end{cases}$	$\begin{cases} 46^{\circ}37' 29.15", \\ 29^{\circ}29' 35.41", \\ 30^{\circ}22' 39.48". \end{cases}$	$\begin{cases} 44^{\circ}58' 17.27", \\ 29^{\circ}36' 31.07", \\ 28^{\circ}43' 27.60". \end{cases}$	27.04064	(92) to (96) exemplify the process of finding the tetrahedron of maximum volume when $a-x$, $b+y$, c , and z are given.
94.	$\begin{cases} a=7.0871, \\ b=5.9769, \\ c=6.1035. \end{cases}$ $\begin{cases} x=6.6870, \\ y=5.9500, \\ z=5.7634. \end{cases}$	$\begin{cases} 33^{\circ}24' 13.34", \\ 26^{\circ}24' 9.16", \\ 25^{\circ}45' 7.03". \end{cases}$	$\begin{cases} 34^{\circ}48' 5.32", \\ 27^{\circ}7' 42.21", \\ 27^{\circ}48' 9.75". \end{cases}$	$\begin{cases} 37^{\circ}7' 10.76", \\ 28^{\circ}15' 20.69", \\ 28^{\circ}57' 40.82". \end{cases}$	$\begin{cases} 45^{\circ}52' 4.32", \\ 29^{\circ}58' 33.41", \\ 30^{\circ}49' 22.42". \end{cases}$	$\begin{cases} 44^{\circ}12' 24.45", \\ 30^{\circ}0' 31.53", \\ 29^{\circ}9' 42.55". \end{cases}$	27.11766	
95.	$\begin{cases} a=7.0841, \\ b=5.9807, \\ c=6.1635. \end{cases}$ $\begin{cases} x=6.6840, \\ y=5.9462, \\ z=5.7634. \end{cases}$	$\begin{cases} 33^{\circ}23' 1.17", \\ 26^{\circ}23' 32.04", \\ 25^{\circ}45' 19.88". \end{cases}$	$\begin{cases} 34^{\circ}47' 49.00", \\ 27^{\circ}9' 27.60", \\ 27^{\circ}49' 4.82". \end{cases}$	$\begin{cases} 37^{\circ}5' 12.52", \\ 28^{\circ}14' 41.58", \\ 28^{\circ}57' 44.80". \end{cases}$	$\begin{cases} 45^{\circ}49' 45.68", \\ 30^{\circ}0' 48.71", \\ 30^{\circ}50' 36.71". \end{cases}$	$\begin{cases} 44^{\circ}10' 15.62", \\ 30^{\circ}0' 52.12", \\ 29^{\circ}11' 4.12". \end{cases}$	27.117785	
96.	$\begin{cases} a=7.0841, \\ b=5.9808, \\ c=6.1635. \end{cases}$ $\begin{cases} x=6.6840, \\ y=5.9468, \\ z=5.7634. \end{cases}$	$\begin{cases} 33^{\circ}23' 0.26", \\ 26^{\circ}23' 29.95", \\ 25^{\circ}45' 19.18". \end{cases}$	$\begin{cases} 34^{\circ}47' 50.76", \\ 27^{\circ}9' 59.52", \\ 27^{\circ}49' 5.29". \end{cases}$	$\begin{cases} 37^{\circ}5' 11.30", \\ 28^{\circ}14' 39.36", \\ 28^{\circ}57' 43.86". \end{cases}$	$\begin{cases} 45^{\circ}49' 46.14", \\ 30^{\circ}0' 50.02", \\ 30^{\circ}50' 36.16". \end{cases}$	$\begin{cases} 44^{\circ}10' 13.86", \\ 30^{\circ}0' 50.02", \\ 29^{\circ}11' 3.88". \end{cases}$	27.117785	
97.	$\begin{cases} a=4.886136, \\ b=3.967768, \\ c=4.3557. \end{cases}$ $\begin{cases} x=3.775036, \\ y=3.572532, \\ z=3.2146. \end{cases}$	$\begin{cases} 26^{\circ}58' 18.90", \\ 22^{\circ}33' 53.45", \\ 20^{\circ}28' 30.21". \end{cases}$	$\begin{cases} 33^{\circ}40' 27.35", \\ 26^{\circ}59' 59.41", \\ 29^{\circ}33' 10.32". \end{cases}$	$\begin{cases} 42^{\circ}31' 24.38", \\ 30^{\circ}35' 32.63", \\ 33^{\circ}36' 15.88". \end{cases}$	$\begin{cases} 48^{\circ}26' 47.83", \\ 30^{\circ}14' 47.50", \\ 33^{\circ}41' 35.39". \end{cases}$	$\begin{cases} 41^{\circ}33' 12.17", \\ 30^{\circ}14' 47.56", \\ 26^{\circ}47' 59.73". \end{cases}$	6.445679	$A-X=C-Z, B+Y=C+Z, A+X=180, B-Y=0$, a similar case of maximum volume.
98.	$\begin{cases} a=5.3147, \\ b=4.0357, \\ c=4.3557. \end{cases}$ $\begin{cases} x=4.2036, \\ y=3.5146, \\ z=3.2146. \end{cases}$	$\begin{cases} 30^{\circ}9' 27.67", \\ 20^{\circ}35' 27.40", \\ 18^{\circ}33' 42.30". \end{cases}$	$\begin{cases} 38^{\circ}35' 14.65", \\ 24^{\circ}32' 54.36", \\ 27^{\circ}4' 4.68". \end{cases}$	$\begin{cases} 49^{\circ}53' 23.33", \\ 31^{\circ}41' 28.99", \\ 31^{\circ}41' 24.26". \end{cases}$	$\begin{cases} 58^{\circ}15' 33.72", \\ 23^{\circ}29' 14.39", \\ 26^{\circ}38' 44.13". \end{cases}$	$\begin{cases} 52^{\circ}39' 1.48", \\ 23^{\circ}29' 14.39", \\ 21^{\circ}2' 11.89". \end{cases}$	5.912593	$A-X=C-Z, B+Y=C+Z$
99.	$\begin{cases} a=11.99015, \\ b=12.0002, \\ c=5.01087. \end{cases}$ $\begin{cases} x=10.96995, \\ y=9.7858, \\ z=10.70947. \end{cases}$	$\begin{cases} 23^{\circ}7' 58.6", \\ 20^{\circ}17' 26.61", \\ 26^{\circ}30' 47.00". \end{cases}$	$\begin{cases} 32^{\circ}15' 47.00", \\ 31^{\circ}46' 0.51", \\ 43^{\circ}16' 16.71". \end{cases}$	$\begin{cases} 31^{\circ}43' 12.49", \\ 29^{\circ}49' 11.31", \\ 40^{\circ}41' 11.19". \end{cases}$	$\begin{cases} 31^{\circ}35' 17.82", \\ 37^{\circ}10' 35", \\ 49^{\circ}40' 40.22". \end{cases}$	$\begin{cases} 31^{\circ}35' 16.83", \\ 26^{\circ}43' 47.56", \\ 40^{\circ}19' 19.78". \end{cases}$	166.9554	Tetrahedron of max. vol. when $a+x, b+y$, and $c-z$ are given.
100.	$\begin{cases} a=5.966907, \\ b=6.028293, \\ c=7.5063. \end{cases}$ $\begin{cases} x=5.4836, \\ y=4.8914, \\ z=4.3556. \end{cases}$	$\begin{cases} 23^{\circ}6' 38.24", \\ 20^{\circ}17' 45.64", \\ 21^{\circ}17' 22.23". \end{cases}$	$\begin{cases} 36^{\circ}14' 24.17", \\ 34^{\circ}43' 53.23", \\ 43^{\circ}26' 9.60". \end{cases}$	$\begin{cases} 33^{\circ}58' 44.51", \\ 29^{\circ}8' 23.94", \\ 40^{\circ}37' 17.00". \end{cases}$	$\begin{cases} 34^{\circ}43' 13.30", \\ 31^{\circ}43' 13.30", \\ 46^{\circ}38' 17.92". \end{cases}$	$\begin{cases} 36^{\circ}13' 46.82", \\ 29^{\circ}36' 59.18", \\ 31^{\circ}39' 31.09". \end{cases}$	18.889605	$a_1=b_1, A=B$

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	VOLUME.	REMARKS.
101. $\begin{cases} a=6.4327, \\ b=6.12078, \\ c=5.7511, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	5.727094	$A+B=180^\circ$ $A+\lambda=B+\lambda=C+Z$
102. $\begin{cases} a=6.4317, \\ b=7.273188, \\ c=3.3423, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	10.22294	$a_4+b_4=180^\circ=A+B$; $A+\lambda=B+\lambda$; $A-\lambda=C-Z$
103. $\begin{cases} a=5.8311, \\ b=5.2931, \\ c=6.3758, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	7.895052	$a_4+b_4=180^\circ=A+Z$; $A+\lambda=B+\lambda$; $A-\lambda=C-Z$
104. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	10.16938	$a_4+b_4=180^\circ=A+Z$; $A+\lambda=B+\lambda$; $A-\lambda=C-Z$
105. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	10.17433	(105) to (134) are a set of 30 tetrahedra; the same six lengths differently arranged are the edges of each. They here form 5 groups of 6 each, the longest edge having the same opposite edge throughout each group.
106. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	10.15133	(105) to (134) are a set of 30 tetrahedra; the same six lengths differently arranged are the edges of each. They here form 5 groups of 6 each, the longest edge having the same opposite edge throughout each group.
107. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	9.9560565	(105) to (134) are a set of 30 tetrahedra; the same six lengths differently arranged are the edges of each. They here form 5 groups of 6 each, the longest edge having the same opposite edge throughout each group.
108. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	9.971434	(105) to (134) are a set of 30 tetrahedra; the same six lengths differently arranged are the edges of each. They here form 5 groups of 6 each, the longest edge having the same opposite edge throughout each group.
109. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	10.103635	(105) to (134) are a set of 30 tetrahedra; the same six lengths differently arranged are the edges of each. They here form 5 groups of 6 each, the longest edge having the same opposite edge throughout each group.
110. $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941, \end{cases}$	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}d$	$\frac{1}{2}e$	$\frac{1}{2}f$	$\frac{1}{2}g$	$\frac{1}{2}h$	$\frac{1}{2}i$	$\frac{1}{2}j$	$\frac{1}{2}k$	$\frac{1}{2}l$	$\frac{1}{2}m$	$\frac{1}{2}n$	$\frac{1}{2}o$	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}r$	$\frac{1}{2}s$	$\frac{1}{2}t$	$\frac{1}{2}u$	$\frac{1}{2}v$	$\frac{1}{2}w$	$\frac{1}{2}x$	$\frac{1}{2}y$	$\frac{1}{2}z$	10.00527	In (110), (113) the volumes are equal, and the values of B supplementary.

EXAMPLES. X.—Continued.

DATA.	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	VOLUME.	REMARKS.
$\begin{matrix} - \\ \text{---} \\ \text{---$																	

EXAMPLES. X.—Continued.

DATA.	$\delta\alpha_1$	$\delta\beta_1$	$\delta\gamma_1$	$\delta\alpha_2$	$\delta\beta_2$	$\delta\gamma_2$	$\delta\alpha_3$	$\delta\beta_3$	$\delta\gamma_3$	δA	δB	δC	$\delta A, \delta B, \delta C$	VOLUME.	REMARKS.
171. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
172. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
173. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
174. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
175. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
176. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
177. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
178. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
179. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	
180. { a=9.4106, b=8.5602, c=4.8370, x=8.0746, y=6.1874, z=7.5278	33°49'51.15"	35°41'13.44"	35°38'4.95"	58°20'29.25"	58°20'29.25"	58°20'29.25"	35°38'4.95"	35°38'4.95"	35°38'4.95"	61°19'11.26"	54°43'24.15"	29°30'8.66"	18°23'44.40"	27.25348	

See page 48.

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$	$\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$	$\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$	$\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$	$\frac{1}{2}a_5, \frac{1}{2}b_5, \frac{1}{2}c_5$	$\frac{1}{2}a_6, \frac{1}{2}b_6, \frac{1}{2}c_6$	$\frac{1}{2}a_7, \frac{1}{2}b_7, \frac{1}{2}c_7$	$\frac{1}{2}a_8, \frac{1}{2}b_8, \frac{1}{2}c_8$	$\frac{1}{2}a_9, \frac{1}{2}b_9, \frac{1}{2}c_9$	$\frac{1}{2}a_{10}, \frac{1}{2}b_{10}, \frac{1}{2}c_{10}$	$\frac{1}{2}a_{11}, \frac{1}{2}b_{11}, \frac{1}{2}c_{11}$	$\frac{1}{2}a_{12}, \frac{1}{2}b_{12}, \frac{1}{2}c_{12}$	VOLUME.	REMARKS.
191. $\begin{cases} a=3.2480, \\ b=11.5857, \\ c=6.0561, \end{cases}$ $\begin{cases} x=9.7425 \\ y=11.4043 \\ z=5.616 \end{cases}$	$28^\circ 36' 24.94''$ $29^\circ 36' 35.36''$ $22^\circ 20' 4.42''$	$27^\circ 15' 41.69''$ $29^\circ 12' 20.80''$ $13^\circ 34' 17.34''$	$38^\circ 27' 34.78''$ $36^\circ 12' 36.48''$ $15^\circ 45' 11.24''$	$40^\circ 49' 7.30''$ $45^\circ 38' 23.82''$ $26^\circ 31' 41.83''$	$46^\circ 47' 40.05''$ $44^\circ 25' 57.77''$ $16^\circ 11' 15.28''$	$36^\circ 27' 6.17''$ $38^\circ 48' 48.45''$ $26^\circ 31' 49.16''$							89.43556	See previous page.
192. $\begin{cases} a=4.1773, \\ b=3.8245, \\ c=3.1189, \end{cases}$ $\begin{cases} x=3.2088 \\ y=3.5616 \\ z=2.8560 \end{cases}$	$26^\circ 56' 50.38''$ $28^\circ 6' 14.81''$ $20^\circ 45' 52.88''$	$29^\circ 23' 20.47''$ $31^\circ 18' 4.55''$ $23^\circ 20' 58.78''$	$37^\circ 56' 2.57''$ $35^\circ 49' 42.50''$ $25^\circ 52' 27.07''$	$38^\circ 32' 46.41''$ $37^\circ 10' 42.55''$ $24^\circ 46' 57.03''$	$43^\circ 6' 11.78''$ $40^\circ 9' 33.98''$ $26^\circ 57' 7.56''$	$35^\circ 51' 19.78''$ $38^\circ 47' 57.58''$ $25^\circ 35' 31.16''$							4.344699	
193. $\begin{cases} a=4.1773, \\ b=3.8245, \\ c=3.1189, \end{cases}$ $\begin{cases} x=3.2088 \\ y=2.8560 \\ z=3.5616 \end{cases}$	$26^\circ 56' 50.38''$ $21^\circ 31' 30.82''$ $26^\circ 19' 2.30''$	$29^\circ 23' 20.47''$ $29^\circ 17' 39.70''$ $24^\circ 8' 24.41''$	$34^\circ 23' 18.00''$ $24^\circ 16' 57.03''$ $25^\circ 52' 27.07''$	$44^\circ 18' 4.77''$ $37^\circ 10' 42.55''$ $35^\circ 49' 42.50''$	$46^\circ 53' 48.22''$ $36^\circ 59' 13.17''$ $28^\circ 36' 4.57''$	$37^\circ 11' 24.76''$ $27^\circ 16' 49.71''$ $35^\circ 39' 58.31''$							4.344699	
194. $\begin{cases} a=4.1773, \\ b=3.8245, \\ c=3.1189, \end{cases}$ $\begin{cases} x=3.2088 \\ y=2.8560 \\ z=3.5616 \end{cases}$	$25^\circ 40' 26.75''$ $23^\circ 20' 58.78''$ $20^\circ 45' 52.88''$	$32^\circ 23' 43.67''$ $31^\circ 18' 4.55''$ $28^\circ 6' 14.81''$	$37^\circ 56' 2.57''$ $30^\circ 47' 8.57''$ $30^\circ 24' 4.17''$	$38^\circ 32' 46.41''$ $34^\circ 16' 49.08''$ $26^\circ 31' 41.83''$	$44^\circ 44' 42.67''$ $30^\circ 21' 47.06''$ $31^\circ 40' 46.96''$	$38^\circ 20' 24.06''$ $32^\circ 32' 5.25''$ $27^\circ 51' 5.15''$							4.452748	Another system of six tetrahedra, having similar relations to those stated for the set (186) to (191).
195. $\begin{cases} a=4.1773, \\ b=3.8245, \\ c=3.1189, \end{cases}$ $\begin{cases} x=3.2088 \\ y=2.8560 \\ z=3.5616 \end{cases}$	$25^\circ 40' 26.75''$ $21^\circ 11' 35.30''$ $22^\circ 48' 30.09''$	$32^\circ 23' 43.67''$ $30^\circ 36' 11.57''$ $28^\circ 36' 20.47''$	$37^\circ 56' 2.57''$ $26^\circ 49' 7.76''$ $30^\circ 24' 4.17''$	$34^\circ 16' 49.08''$ $34^\circ 16' 49.08''$ $30^\circ 47' 8.57''$	$45^\circ 15' 17.33''$ $35^\circ 21' 56.22''$ $32^\circ 26' 53.18''$	$38^\circ 31' 32.32''$ $28^\circ 38' 11.21''$ $31^\circ 33' 14.25''$							4.452748	
196. $\begin{cases} a=4.1773, \\ b=3.8245, \\ c=3.1189, \end{cases}$ $\begin{cases} x=3.2088 \\ y=3.5616 \\ z=3.1189 \end{cases}$	$27^\circ 34' 7.34''$ $28^\circ 36' 20.47''$ $22^\circ 48' 30.09''$	$28^\circ 28' 30.67''$ $30^\circ 36' 11.57''$ $21^\circ 11' 35.30''$	$36^\circ 35' 15.34''$ $34^\circ 14' 42.79''$ $23^\circ 27' 22.35''$	$40^\circ 12' 4.23''$ $38^\circ 58' 30.31''$ $27^\circ 16' 46.54''$	$45^\circ 21' 24.63''$ $40^\circ 59' 23.23''$ $24^\circ 48' 27.08''$	$35^\circ 38' 23.94''$ $38^\circ 0' 25.32''$ $27^\circ 47' 25.01''$							4.327226	
197. $\begin{cases} a=4.1773, \\ b=3.8245, \\ c=3.1189, \end{cases}$ $\begin{cases} x=3.2088 \\ y=3.5616 \\ z=3.5616 \end{cases}$	$27^\circ 34' 7.34''$ $24^\circ 8' 24.41''$ $26^\circ 19' 2.30''$	$28^\circ 28' 30.67''$ $29^\circ 17' 39.70''$ $31^\circ 33' 30.82''$	$34^\circ 23' 18.00''$ $27^\circ 16' 46.34''$ $23^\circ 27' 22.35''$	$44^\circ 18' 4.77''$ $38^\circ 58' 30.31''$ $34^\circ 14' 42.79''$	$46^\circ 38' 35.38''$ $38^\circ 47' 48.65''$ $25^\circ 43' 45.48''$	$36^\circ 47' 17.47''$ $30^\circ 22' 42.41''$ $34^\circ 8' 51.72''$							4.327226	
198. $\begin{cases} a=7.39443, \\ b=6.52081, \\ c=4.77357, \end{cases}$ $\begin{cases} x=6.5208 \\ y=7.39442 \\ z=5.64718 \end{cases}$	$34^\circ 15' 52.46''$ $35^\circ 35' 3.59''$ $23^\circ 41' 11.82''$	$29^\circ 5' 4.17''$ $29^\circ 5' 4.24''$ $18^\circ 49' 52.42''$	$37^\circ 13' 43.94''$ $37^\circ 13' 43.87''$ $21^\circ 28' 14.65''$	$33^\circ 55' 3.99''$ $34^\circ 52' 56.77''$ $23^\circ 41' 11.96''$	$41^\circ 31' 55.82''$ $38^\circ 53' 19.58''$ $22^\circ 19' 43.09''$	$38^\circ 53' 19.58''$ $41^\circ 31' 55.63''$ $25^\circ 17' 58.95''$							26.10944	Shortest distance between ϵ and z = 5.840578.
199. $\begin{cases} a=7.39443, \\ b=4.77357, \\ c=6.52081, \end{cases}$ $\begin{cases} x=6.5208 \\ y=4.77357 \\ z=5.64718 \end{cases}$	$34^\circ 15' 52.46''$ $31^\circ 55' 6.33''$ $24^\circ 53' 45.80''$	$29^\circ 5' 4.17''$ $30^\circ 6' 14.20''$ $26^\circ 9' 47.06''$	$45^\circ 0' 0.00''$ $37^\circ 13' 43.87''$ $34^\circ 52' 56.77''$	$31^\circ 55' 6.61''$ $22^\circ 28' 14.68''$ $23^\circ 41' 11.96''$	$48^\circ 8' 24.22''$ $24^\circ 29' 21.10''$ $33^\circ 50' 8.67''$	$41^\circ 35' 26.71''$ $36^\circ 37' 58.64''$ $27^\circ 17' 11.16''$							26.10944	
200. $\begin{cases} a=7.39443, \\ b=6.52081, \\ c=7.39442, \end{cases}$ $\begin{cases} x=6.5208 \\ y=4.77357 \\ z=5.64718 \end{cases}$	$27^\circ 43' 47.22''$ $18^\circ 49' 52.47''$ $23^\circ 41' 11.82''$	$38^\circ 29' 3.70''$ $29^\circ 5' 4.24''$ $35^\circ 35' 3.59''$	$37^\circ 13' 43.94''$ $22^\circ 42' 52.63''$ $34^\circ 52' 25.33''$	$35^\circ 35' 3.99''$ $28^\circ 46' 3.67''$ $26^\circ 46' 3.67''$	$44^\circ 14' 24.21''$ $30^\circ 13' 46.74''$ $37^\circ 34' 15.53''$	$43^\circ 1' 49.46''$ $23^\circ 51' 57.98''$ $31^\circ 30' 26.77''$							26.65186	See next page.

EXAMPLES. X.—Continued.

DATA.	$\frac{1}{2}a$	$\frac{1}{2}b$	$\frac{1}{2}c$	$\frac{1}{2}A$	$\frac{1}{2}B$	$\frac{1}{2}C$	$\frac{1}{2}A$	$\frac{1}{2}B$	$\frac{1}{2}C$	$\frac{1}{2}A$	$\frac{1}{2}B$	$\frac{1}{2}C$	VOLUME.	REMARKS.
211. $\begin{cases} a=7.10142, x=5.618158 \\ b=8.5445, y=6.6434 \\ c=7.6147, z=5.7136 \end{cases}$	$20^{\circ} 5' 13.98''$	$25^{\circ} 41' 16.43''$	$20^{\circ} 22' 37.42''$	$26^{\circ} 43' 18.50''$	$35^{\circ} 53' 22.25''$	$31^{\circ} 47' 6.53''$	$30^{\circ} 52' 38.84''$	$32^{\circ} 31' 37.04''$	$33^{\circ} 36' 44.42''$	$27^{\circ} 35' 20.32''$	$46^{\circ} 56' 1.00''$	$41^{\circ} 54' 19.94''$	35.00578	$\begin{aligned} A+B+C &= 2\pi \\ B-A &= C-Z \\ \sigma_2 &= .97158.71'' \end{aligned}$
212. $\begin{cases} a=9.31534, x=5.06584 \\ b=8.9763, y=4.7268 \\ c=8.01066, z=5.69214 \end{cases}$	$17^{\circ} 3' 4.15''$	$15^{\circ} 14' 45.59''$	$18^{\circ} 6' 6.27''$	$28^{\circ} 38' 34.36''$	$34^{\circ} 19' 35.22''$	$29^{\circ} 39' 17.25''$	$37^{\circ} 34' 18.51''$	$45^{\circ} 5' 57.16''$	$46^{\circ} 14' 22.54''$	$31^{\circ} 51' 31.23''$	$41^{\circ} 30' 0.62''$	$27^{\circ} 7' 9.31''$	30	$\begin{aligned} A-B &= C-Z \\ B+A &= C+Z \end{aligned}$
213. $\begin{cases} a=9.8933, x=5.6138 \\ b=8.9763, y=4.7268 \\ c=8.0107, z=5.6924 \end{cases}$	$19^{\circ} 8' 23.15''$	$14^{\circ} 5' 38.70''$	$17^{\circ} 20' 32.92''$	$32^{\circ} 24' 40.59''$	$31^{\circ} 54' 21.84''$	$26^{\circ} 35' 27.42''$	$40^{\circ} 15' 5.24''$	$49^{\circ} 18' 53.88''$	$37^{\circ} 34' 48.07''$	$38^{\circ} 31' 35.02''$	$29^{\circ} 40' 34.41''$	$23^{\circ} 56' 9.25''$	31.06101	$\begin{aligned} A-B &= C-Z \\ B+A &= C+Z \end{aligned}$
214. $\begin{cases} a=9.8671, x=5.6176 \\ b=8.9763, y=4.7268 \\ c=8.1807, z=5.5224 \end{cases}$	$18^{\circ} 56' 0.90''$	$14^{\circ} 13' 32.89''$	$16^{\circ} 49' 32.50''$	$32^{\circ} 58' 16.63''$	$32^{\circ} 8' 36.50''$	$27^{\circ} 46' 14.72''$	$41^{\circ} 1' 31.00''$	$48^{\circ} 0' 12.39''$	$37^{\circ} 10' 15.00''$	$38^{\circ} 41' 44.17''$	$30^{\circ} 53' 36.10''$	$30^{\circ} 53' 45.39''$	31.12113	$\begin{aligned} A+B &= C-Z \\ A+B &= C+Z \end{aligned}$ in all four; in (216) $A+B=180^{\circ}$, $C=Z$, so that this is the tetrahedron of maxi- mum volume under the conditions.
215. $\begin{cases} a=9.86936, x=5.61986 \\ b=8.9763, y=4.7268 \\ c=8.18092, z=5.52218 \end{cases}$	$18^{\circ} 56' 29.39''$	$14^{\circ} 13' 16.98''$	$16^{\circ} 49' 38.57''$	$32^{\circ} 59' 15.00''$	$32^{\circ} 8' 1.60''$	$27^{\circ} 45' 35.96''$	$41^{\circ} 2' 19.83''$	$48^{\circ} 1' 7.06''$	$37^{\circ} 9' 7.50''$	$38^{\circ} 43' 31.69''$	$30^{\circ} 52' 42.75''$	$30^{\circ} 52' 42.81''$	31.12115	$\begin{aligned} a-x &= b-y = 4.2495, \\ c &= z = 13.7031. \end{aligned}$ $a-x$ is very small ($0^{\circ} 23''$).
216. $\begin{cases} a=9.869433, x=5.619933 \\ b=8.9763, y=4.7268 \\ c=8.180921, z=5.522179 \end{cases}$	$18^{\circ} 56' 30.38''$	$14^{\circ} 13' 16.44''$	$16^{\circ} 49' 38.22''$	$32^{\circ} 59' 16.77''$	$32^{\circ} 8' 0.44''$	$27^{\circ} 45' 34.57''$	$41^{\circ} 2' 21.34''$	$48^{\circ} 1' 8.99''$	$37^{\circ} 9' 5.81''$	$38^{\circ} 43' 35.12''$	$30^{\circ} 52' 40.93''$	$30^{\circ} 52' 40.93''$	31.12115	$\begin{aligned} I &= I_2 = I_3 \\ A_1 + B_1 &= 180^{\circ} \\ H_1 + J_1 &= 180^{\circ} \\ B_1 - J_1 &= 0 \end{aligned}$
217. $\begin{cases} a=9.0481, x=8.8901 \\ b=8.9217, y=9.0797 \\ c=6.4025, z=6.3040 \end{cases}$	$34^{\circ} 20' 5.56''$	$34^{\circ} 47' 28.38''$	$20^{\circ} 31' 59.27''$	$33^{\circ} 55' 43.65''$	$34^{\circ} 11' 43.75''$	$20^{\circ} 40' 55.67''$	$35^{\circ} 16' 16.08''$	$34^{\circ} 31' 35.95''$	$40^{\circ} 47' 59.34''$	$40^{\circ} 17' 31.62''$	$40^{\circ} 59' 25.55''$	$41^{\circ} 29' 53.22''$	52.31824	
218. $\begin{cases} a=9.0481, x=8.8901 \\ b=8.9217, y=9.0797 \\ c=6.4025, z=6.3040 \end{cases}$	$34^{\circ} 20' 5.56''$	$34^{\circ} 47' 28.38''$	$20^{\circ} 31' 59.27''$	$33^{\circ} 55' 43.65''$	$34^{\circ} 11' 43.75''$	$20^{\circ} 40' 55.67''$	$35^{\circ} 16' 16.08''$	$34^{\circ} 31' 35.95''$	$40^{\circ} 47' 59.34''$	$40^{\circ} 17' 31.62''$	$40^{\circ} 59' 25.55''$	$41^{\circ} 29' 53.22''$	52.31824	
219. $\begin{cases} a=9.0481, x=8.8901 \\ b=8.9217, y=9.0797 \\ c=6.4025, z=6.3040 \end{cases}$	$22^{\circ} 28' 34.72''$	$31^{\circ} 47' 28.38''$	$20^{\circ} 31' 59.27''$	$33^{\circ} 55' 43.65''$	$34^{\circ} 11' 43.75''$	$20^{\circ} 40' 55.67''$	$35^{\circ} 16' 16.08''$	$34^{\circ} 31' 35.95''$	$40^{\circ} 47' 59.34''$	$40^{\circ} 17' 31.62''$	$40^{\circ} 59' 25.55''$	$41^{\circ} 29' 53.22''$	52.31824	
220. $\begin{cases} a=9.0481, x=8.8901 \\ b=8.9217, y=9.0797 \\ c=6.4040, z=6.4025 \end{cases}$	$31^{\circ} 30' 7.54''$	$34^{\circ} 57' 35.22''$	$20^{\circ} 52' 8.21''$	$33^{\circ} 46' 30.08''$	$34^{\circ} 2' 13.30''$	$20^{\circ} 20' 58.97''$	$35^{\circ} 5' 38.49''$	$34^{\circ} 31' 25.81''$	$40^{\circ} 47' 27.35''$	$40^{\circ} 18' 3.28''$	$40^{\circ} 59' 21.25''$	$22^{\circ} 3' 9.31''$	52.31891	See next page.

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